SNU mini-Higgs workshop, Sept 28, 2017

# On the Validity of Effective Potential and the Precision of Higgs Self Coupling

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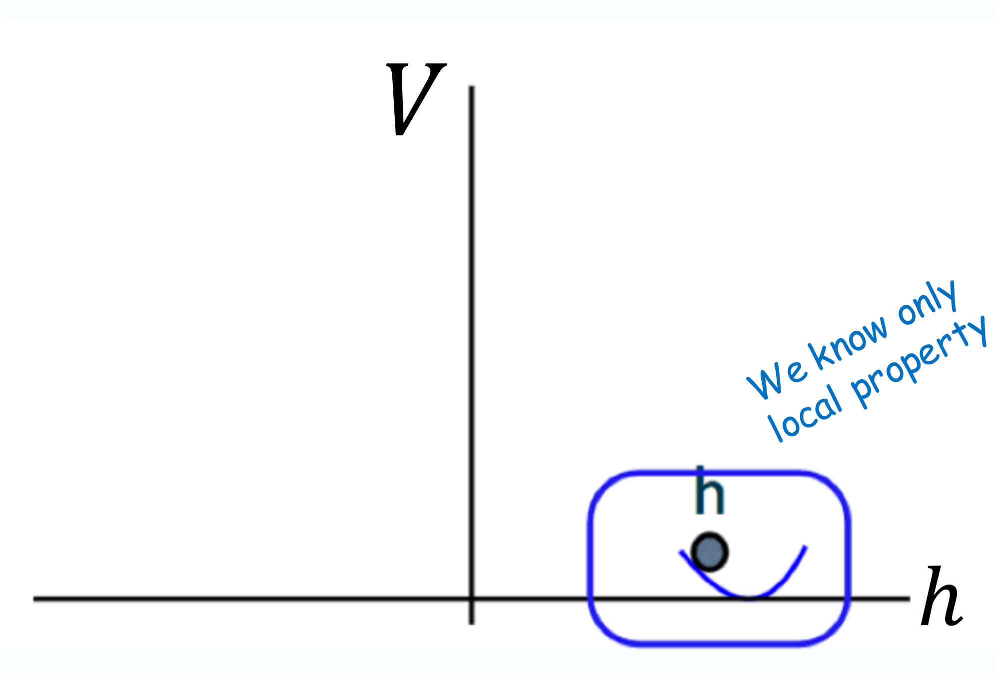
Based on work with S. Lee, Bithika Jain 1709.03232

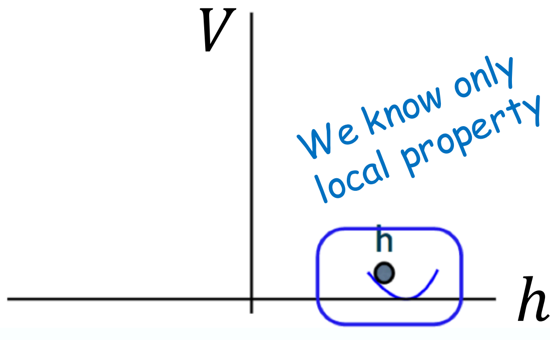
We know VERY little about Higgs potential
in Bottom-up approach

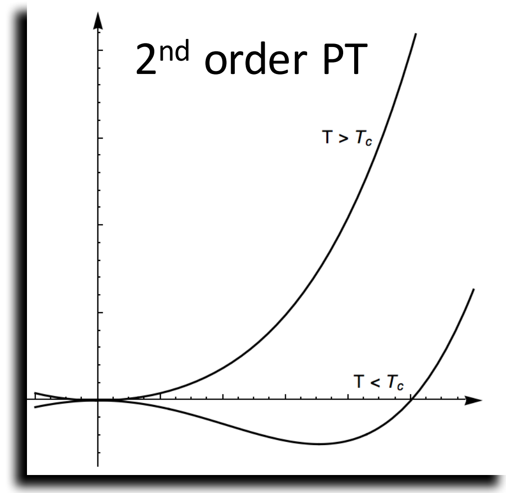



$m_h = 125$ GeV
$v = 246$ GeV

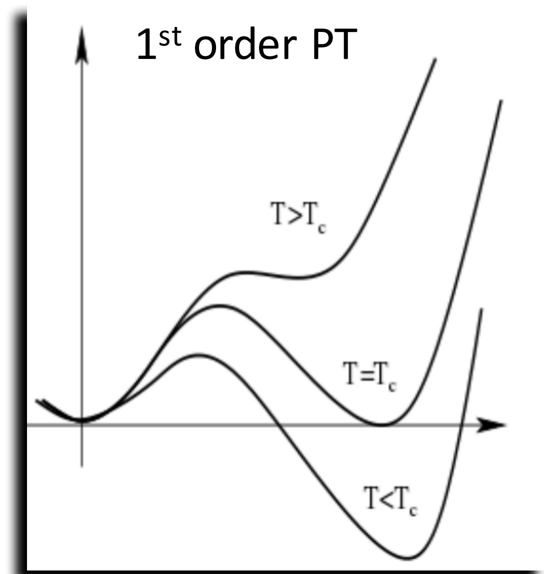
Cartoon stolen
from the talk by Liantao Wang

Various global structure/thermal histories
are still plausible




$$V_h = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6}\left(\frac{3\, m_h^2}{v}\right) h^3 + c_4 \frac{1}{24}\left(\frac{3\, m_h^2}{v^2}\right) h^4$$




An interesting physics still allowed
is baryogenesis based on
strong 1st order EW-phase transition

# Strong 1st order Electroweak Phase Transition

An interesting physics that can be related to the **large** deviation of the Higgs self coupling

# The effective potential

is a main tool to examine the thermal history of the Higgs potential

$$V_{eff} = V_{tree} + V_{CW}[m_i^2(h) + \Pi_i] + V_T[m_i^2(h) + \Pi_i]$$

$$V_T = \sum_{i=\mathrm{B,F}} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} \int_0^\infty dx\, x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2 + {(m_i^2(h) + \Pi_i)}/{T^2}}\right)\right]$$

Obtaining the exact thermal mass is very non-trivial

What most people do is using

We call this Prescription A

Truncated Full Dressing (TFD):

: thermal mass $\Pi_i$ is still obtained in the high-T approximation

At Leading order in Temperature, $\Pi_\mathrm{i}$ is mass-independent

→ non-decoupling issue

: the related uncertainty has not been well understood in BSM scenarios

Curtin, Meade, Ramani 16' for a recent discussion

# The effective potential

In the High-T approximation

We call this **Prescription B**

$$V_{eff} = V_{tree} + V_{CW} + V_T + V_{ring}$$

$$\begin{cases} V_{CW} = \sum_{i=t,W,Z,h,G,\cdots} (-1)^{F_i} \frac{g_i}{64\pi^2}\left[m_i^4(h)\left(\log\frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2}\right) + 2m_i^2(h)m_i^2(v)\right] \\ V_T = \sum_{i=\mathrm{B,F}} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2(h)}{T^2}\right) \\ V_{ring} = \sum_{i=\mathrm{bosons}} \frac{\bar{g}_i T}{12\pi}\left[m_i^3(h) - \left(m_i^2(h) + \Pi_i(T)\right)^{\frac{3}{2}}\right] \end{cases}$$

$$x^2 = \frac{m^2}{T^2} \ll 1$$

1st order PT via thermal effect

$$\boxed{\begin{aligned} J_B(x^2) &\approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 \boxed{- \frac{\pi}{6}x^3} - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_b}\right) \\ J_F(x^2) &\approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_f}\right) \end{aligned}}$$

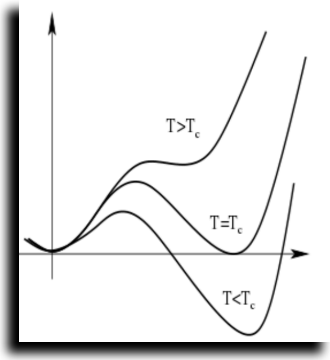
✓ Validity of this approx. carefully has to be checked

# On the criteria for strong 1st order phase transition

: There has been an ambiguity on how to quantify the strong first order phase transition and this ambiguity can cause $\mathcal{O}(1)$ fluctuation on the precision of Higgs self coupling

List of sources that can cause $\mathcal{O}(1)$ uncertainty

$$\frac{m_i^2(v_c)}{T_c^2} = \frac{m^2}{T_c^2} + \text{coupling}\times\frac{v_c^2}{T_c^2} \gtrsim 1$$




For $v_c \gtrsim T_c$ and $\gtrsim \mathcal{O}(1)$ coupling,
integral needs to be exactly evaluated

1. The High-temperature Approximation
2. A Large coupling
3. $\frac{v_c}{T_c}$ vs $\frac{v_N}{T_N}$ , and confusion on $\boxed{\frac{v_c}{T_c} \gtrsim 0.6-1.4}$

: checks if the potential develops degenerate vacua

: checks if the transition actually happens

The impact on the precision of the Higgs self-coupling of these issues has not been well studied in most literature in the context of BSM physics

To illustrate the issues we take

# Most commonly considered frameworks

$$V_{eff} = \sum_{i=t,W,Z,h,G,\boxed{\mathrm{BSM}}} V_i$$

No mixing with Higgs
: less constrained

Mixes with Higgs
: strongly constrained

Higgs portal:

new scalar S

$Z_2$-symmetry, e.g. $\langle S \rangle = 0$ vs $\langle S \rangle \neq 0$

$N_s$: multiplicity E.g. can help with weak coupling

….

Effective Field Theory:

higher-dimensional operators

$\mathcal{O}_H = (\partial |H|^2)^2$ vs $\mathcal{O}_6 = |H|^6$

Higgs decay
: strongly constrained

Higgs self coupling
: poorly constrained

Weakly coupled theory

Strongly coupled theory

PGB vs non-pGB
Higgs Portal

$\mathcal{O}_H \ll \mathcal{O}_6$ possible

Azatov, Contino, Panico, Son 15’

# $\mathcal{O}_H \ll \mathcal{O}_6$ possible

Azatov, Contino, Panico, Son 15'

E.g.

Higgs : p~~G~~B

Generic composite state: tuned …
→ no supurion suppression.

→ Enhancement by $\frac{g_*^2}{g_{\cancel{GB}}}$

$$\bar{c}_H \sim \left(\frac{v}{f}\right)^2 \sim 0.05$$

$$\bar{c}_6 \sim \left(\frac{v}{f}\right)^2 \frac{g_*^2}{\lambda_4} \sim 3.5 \left(\frac{g_*}{3}\right)^2$$

Higgs portal (to strongly coupled sector)

$$\mathcal{L} = \lambda |H|^2 \mathcal{O}$$

$\mathcal{O}$ characterized by $\{m_*, g_*\}$

$$\bar{c}_H \sim \left(\frac{v}{f}\right)^2 \times \frac{\lambda^2}{g_*^4}$$

$$\bar{c}_6 \sim \left(\frac{v}{f}\right)^2 \times \frac{\lambda^3}{g_*^4 \lambda_4}$$

$$\bar{c}_H / \bar{c}_6 = \lambda_4 / \lambda$$

Up to some fine-tuning

Strong 1st order
Electroweak Phase Transition

# Higgs Portal
# with a singlet scalar

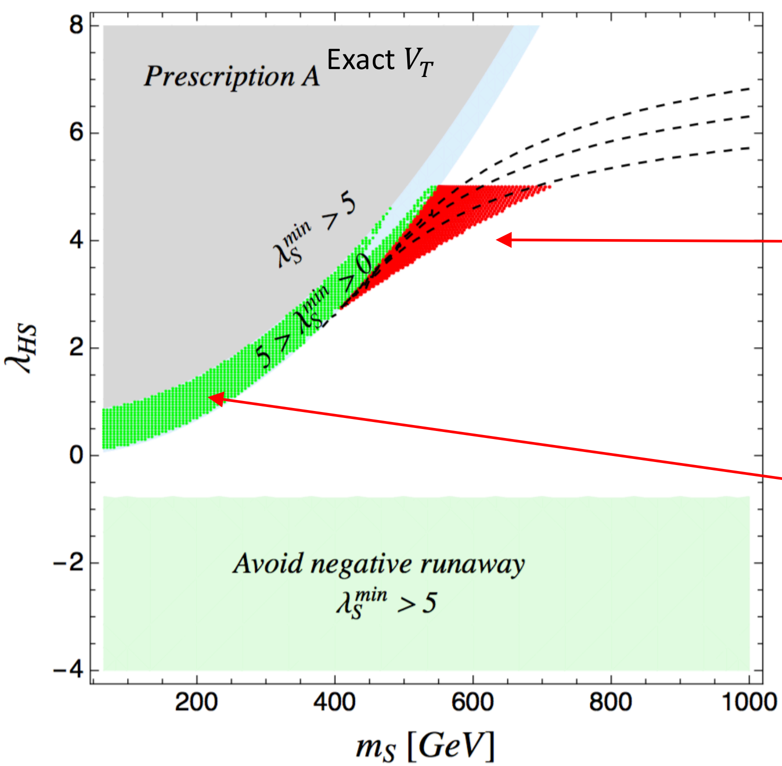
Noble, Perelstein 08'
Katz, Perelstein 14'
Curtin, Meade, Yu 14'
Kurup, Perelstein 17'
Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughness 03'
Espinosa, Konstandin, Riva 11'
Cline, Kaiulainen 12'
Alanne, Tuominen, Vaskonen 14'
Many others ….

# Higgs Portal

SM + a singlet scalar with Z2

$$V_{tree} = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}\lambda_{HS} h^2 S^2 + \frac{1}{2} m_{0\ s}^2 S^2 + \frac{1}{4}\lambda_S S^4$$

$(\langle h\rangle, \langle s\rangle) = (v, 0)$ is a global minimum




Based on naïve criterion, existence of degenerate vacua, with $v_c/T_c > 1$ * Note cutoff $\lambda_{HS} < 5$ by hand

1. One-step strong 1st phase transition (**RED**)

$$V(0,0) \rightarrow V(v,0) \quad , \langle S\rangle = 0$$

2. Two-step strong 1st phase transition (**GREEN**)

$$V(0,0) \rightarrow V(0,v_s) \rightarrow V(v,0)$$

$$V(0,v_s) > V(v,0) \rightarrow$$

$$\lambda_S > \lambda_S^{\min} \equiv \lambda \frac{m_{0\ s}^4}{\mu^4} = \frac{2(m_S^2 - v^2\lambda_{HS})^2}{m_h^2 v^2}$$

parametrize $\lambda_s = \lambda_s^{min} + \delta_s$

Scan:

$m_s = [100,800]$ GeV in steps of 10 GeV

$\lambda_{HS} = [1,5]$ in steps of 0.2

Similar plot in Curtin, Meade, Ramani 14'

# Higgs Portal

SM + a singlet scalar with Z2

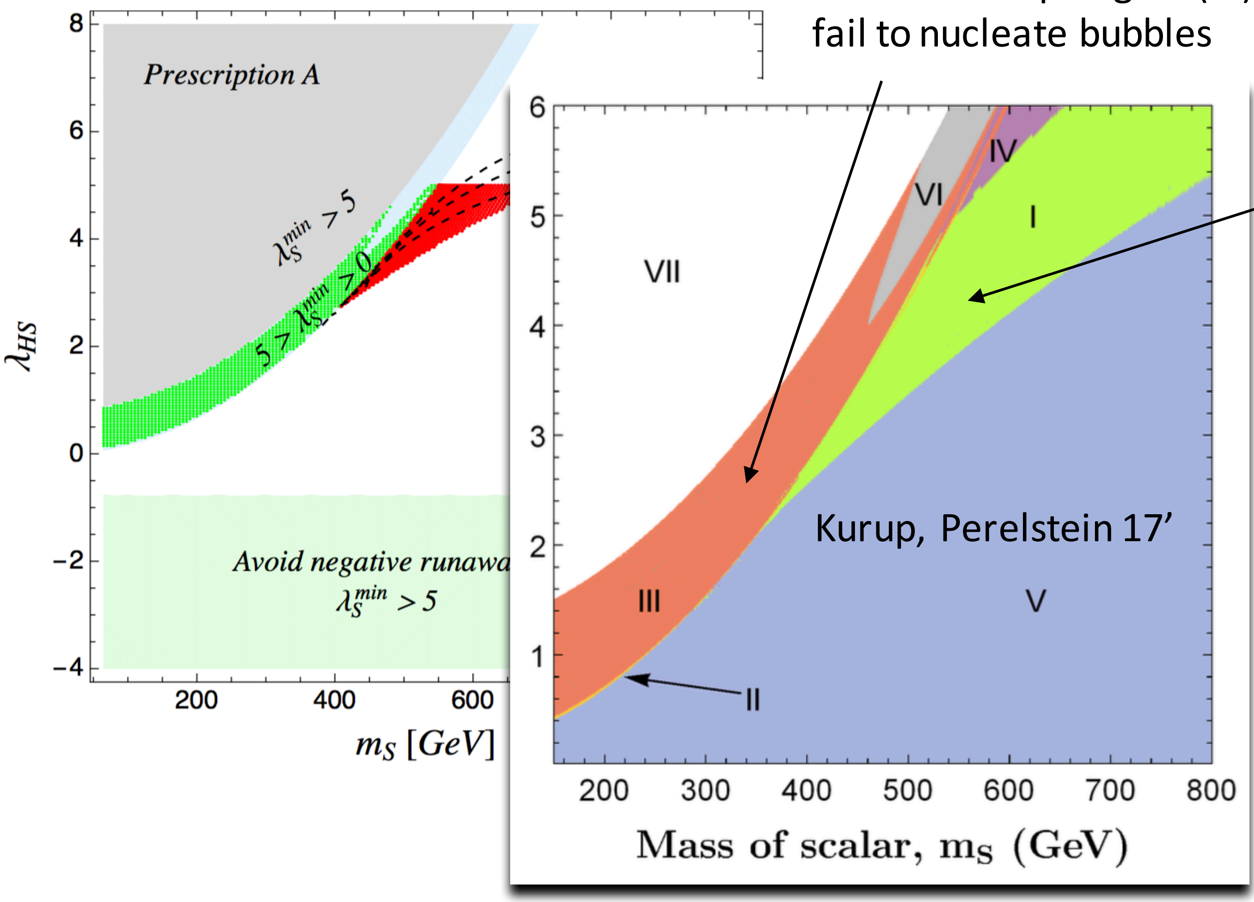
## Fate of two step PT

$(\langle h\rangle, \langle s\rangle) = (v, 0)$ is a global minimum

Most two-step region (III) fail to nucleate bubbles

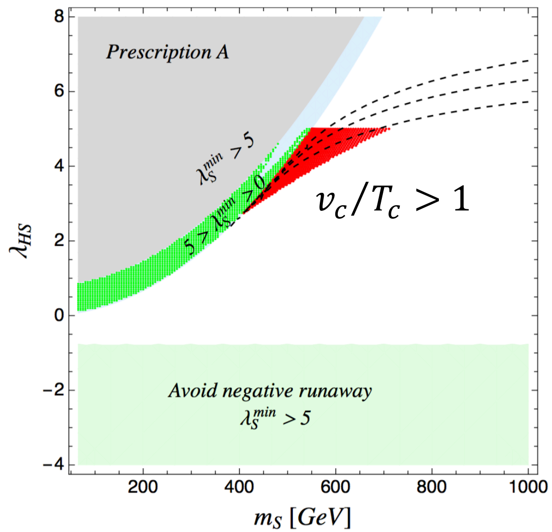
## One step PT

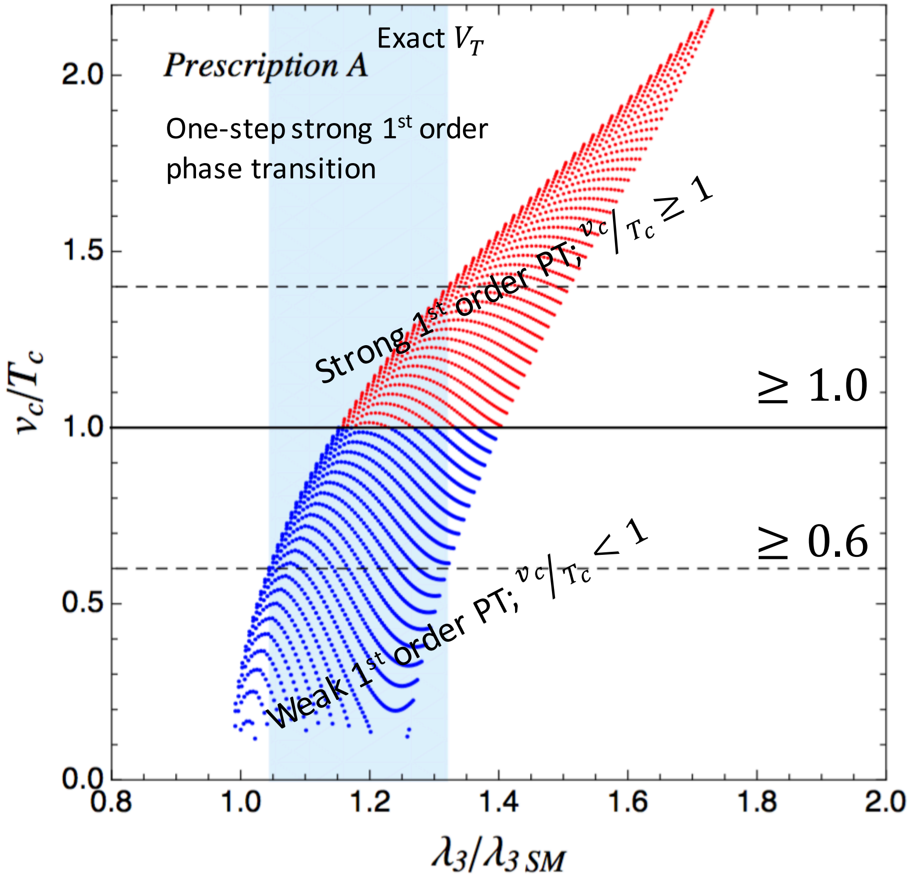
Most region (I) survives : nucleates




We will focus on the one-step PT

$(\langle h\rangle, \langle s\rangle) = (v, 0)$ is a global minimum




$\sim 5\%$ $\sim 32\%$

✓ fluctuates by $\mathcal{O}(1)$ amount

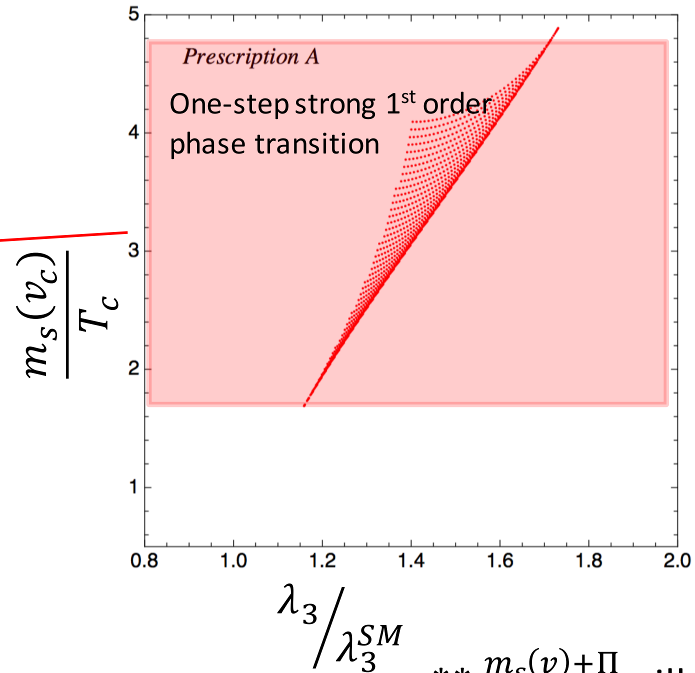
Future collider plan can sensitively depend on the exact criteria

E.g. only 100 TeV pp
vs various colliders 100 TeV, ILC?

## Q. Is there a preferred ${}^{v_c}/_{T_c}$?

Spannowsky, SON, Ye in progress

# Validity of High-T approximation

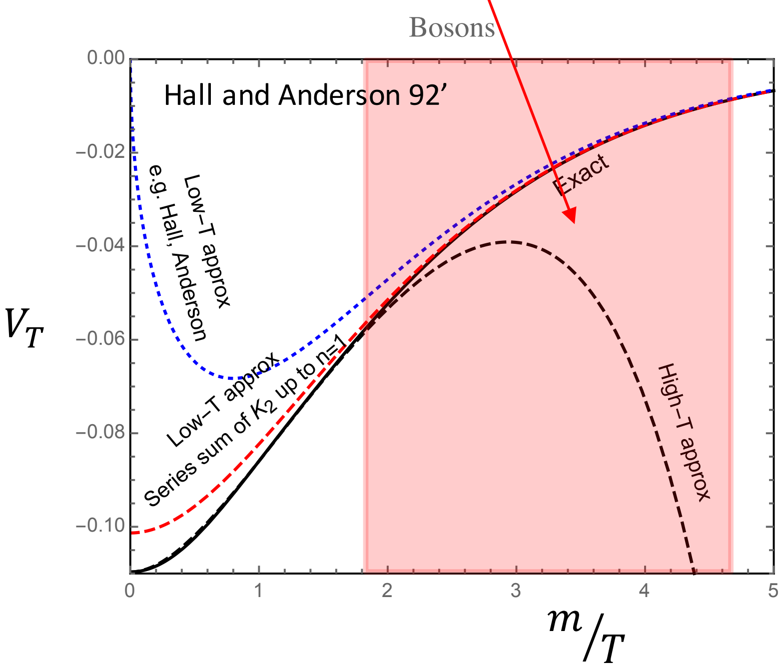



One-step PT

High-T approx. fails

(The issue is more pronounced in two step PT)

$** \ \frac{m_s(v)+\Pi}{T_c}$ will be even bigger




$$x^2 = \frac{m^2}{T^2} \ll 1\text{: High T approximation}$$

$$J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_b}\right)$$

$$J_F(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_f}\right)$$

$$x^2 = \frac{m^2}{T^2} \gg 1\text{: Low T approximation}$$

$$J_B(x^2; n) = -\sum_{k=1}^{n} \frac{1}{k^2} x^2 K_2(x\,k)$$

$$J_F(x^2; n) = -\sum_{k=1}^{n} \frac{(-1)^k}{k^2} x^2 K_2(x\,k)$$

Low-T approx with $n = \mathcal{O}(10)$

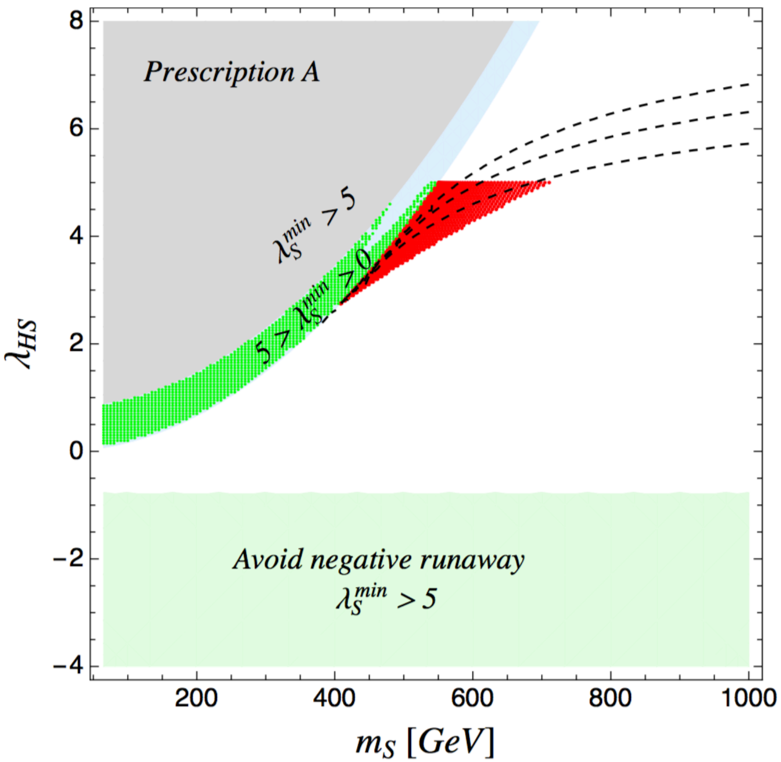
= Exact evaluation, e.g. chose $n = 50$

# Validity of high-T approximation

Prescription A

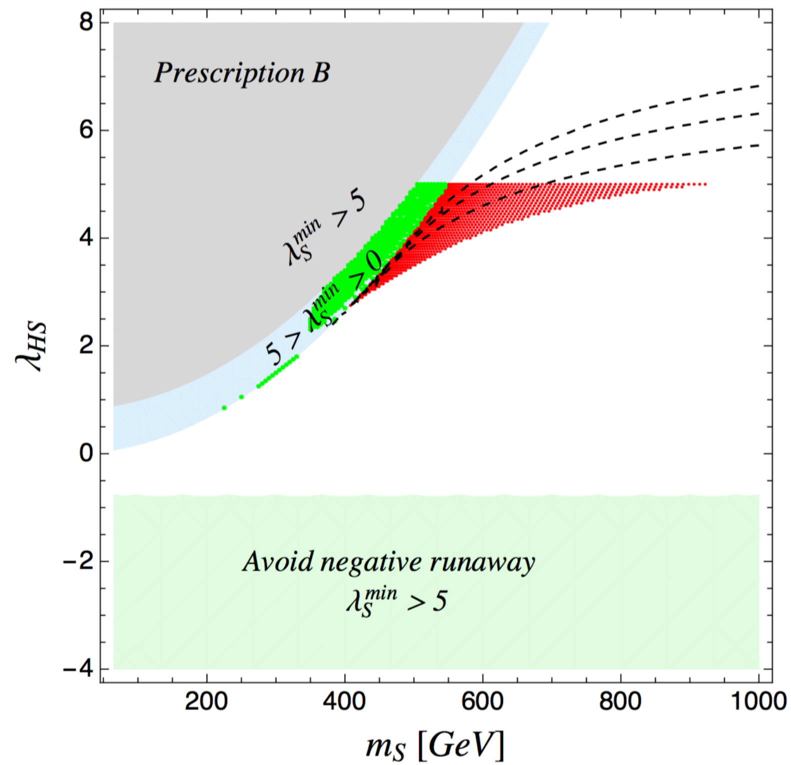
Exact $V_T$

* Note cutoff $\lambda_{HS}$ < 5 by hand




Prescription B

High-T approx $V_T$




The results look similar in these plots

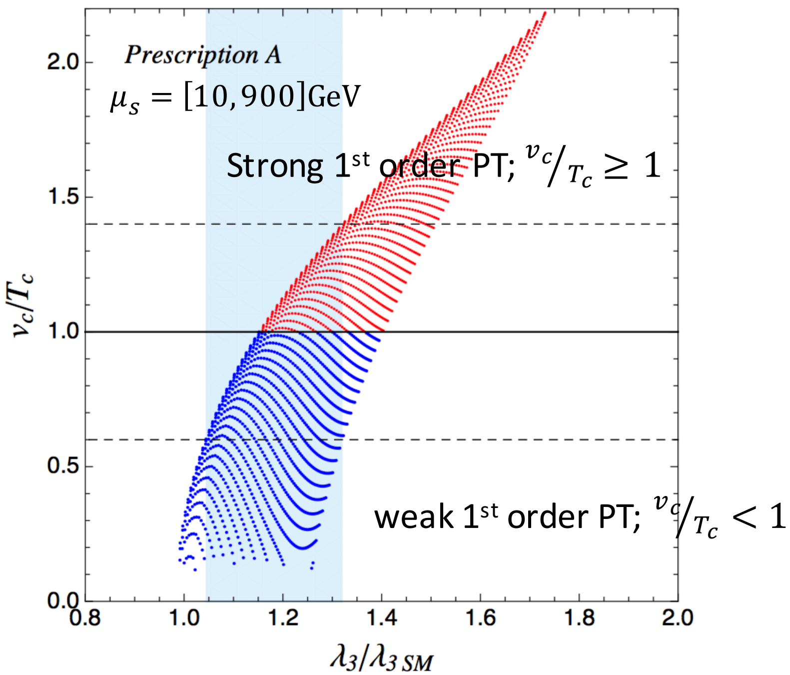
: discrepancy is pronounced in the large coupling region where high-T approximation fails too

# $\mathcal{O}(1)$ fluctuation in Higgs self-coupling

Prescription A

## Exact $V_T$

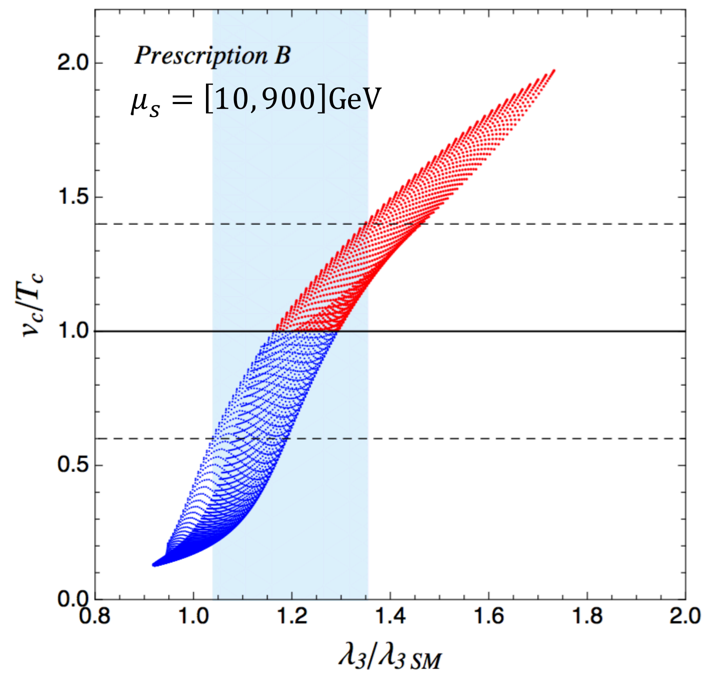
One-step strong 1st order phase transition




Prescription B

## High-T approx $V_T$

One-step strong 1st order phase transition

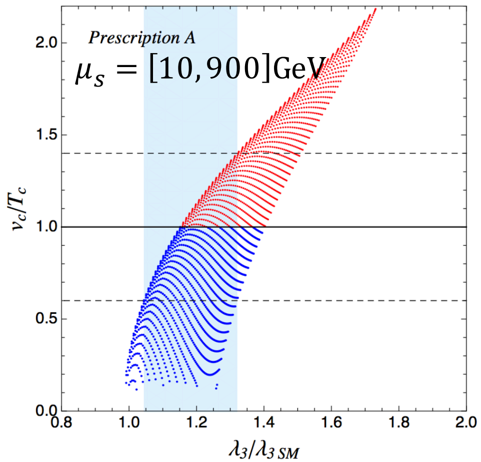



Similarly for both, the smallest deviation, $\delta\left(^{\lambda_3}/_{\lambda_3^{SM}}\right) \sim [5, 32]\%$ for $^{v_c}/_{T_c} \geq [0.6, 1.4]$

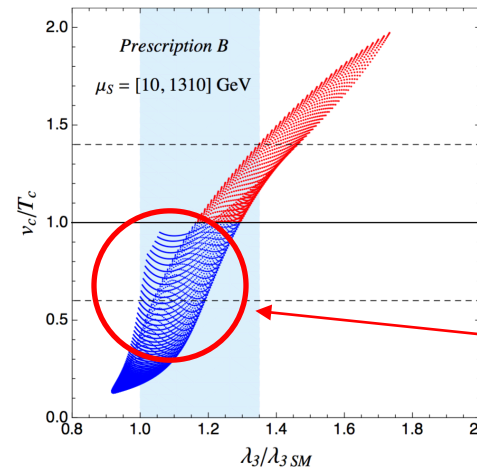
The patterns of parameter space in two cases look very different! What's going on ?

## $\mathcal{O}(1)$ fluctuation in Higgs self-coupling

### Exact $V_T$

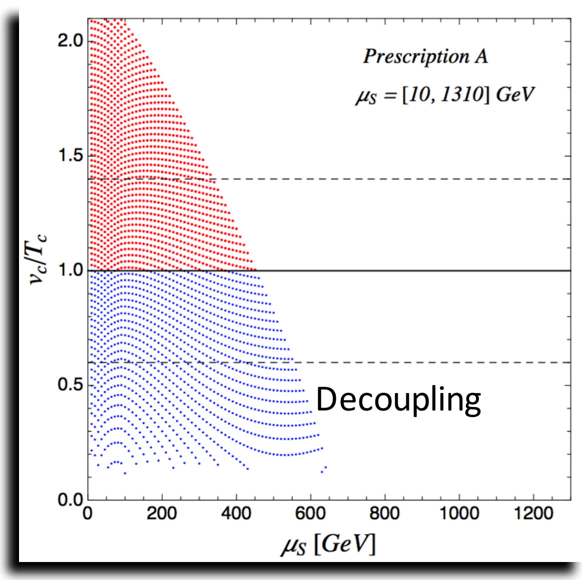


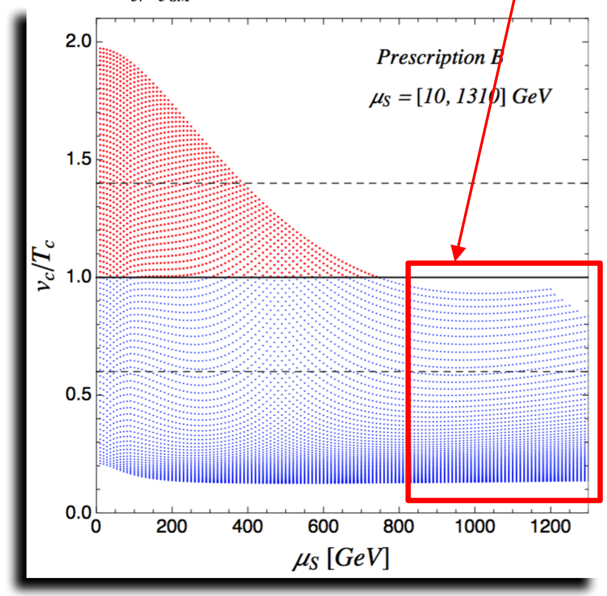

### High-T approx. $V_T$




$\mu_s = [10, 1310]$GeV

High masses do not decouple and screws up Higgs self coupling




High-T approx. does not seem to be appropriate for Higgs portal.

Strong 1st order
Electroweak Phase Transition

# EFT approach

Delaunay, Grojean, Wells, 08'
Huang, Joglekar, Li, Wagner 15'
Chung, Long, Wang 16'
….

# $1^{st}$ order phase transition in EFT approach

** Suitable for the case with $\mathcal{O}_H \ll \mathcal{O}_6$, nevertheless we will present results in SILH basis

Case I: include only dim-6 operator $\mathcal{O}_6 \sim |H|^6$

& assume Higgs as pGB: NDA in SILH basis: $c_6 \sim \left(\frac{v}{f}\right)^2 \equiv \xi$

$$\mathrm{V}_{EFT} = m^2|H|^2 + \lambda|H|^4 + \frac{c_6}{v^2}\frac{m_h^2}{2\,v^2}|H|^6$$

$$\rightarrow \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{8}\,\frac{c_6}{v^2}\frac{m_h^2}{2\,v^2}h^6$$

where $\lambda = \frac{m_h^2}{2v^2}\left(1 - \frac{3}{2}c_6\right),\ \ m^2 = -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c_6\right)$

At tree-level

$$\lambda_3 \equiv \left.\frac{d^3V_{EFT}}{dh^3}\right|_{h=v} = \frac{3m_h^2}{v}(1 + c_6)$$

$$\lambda_4 \equiv \left.\frac{d^4V_{EFT}}{dh^4}\right|_{h=v} = \frac{3m_h^2}{v^2}(1 + 6\,c_6)$$

$$\lambda_3/\lambda_3^{SM} = \boxed{1 + c_6}$$

$$\lambda_4/\lambda_4^{SM} = \boxed{1 + 6\,c_6}$$

Relation holds only at the level of dimension-six

# $1^{st}$ order phase transition in EFT approach

Assuming $m^2(T) = m^2 + aT^2$ (keeping only $T^2$-term as thermal effect) the analytic constraint on $c_6$ to have $1^{st}$ order phase transition is possible

$$\left.\frac{dV_{EFT}}{dh}\right|_{h=v_c,\,T=T_c} = 0$$ stationary condition

$$V(v_c, T_c) = V(0, T_c)$$ degeneracy of the vacua

$$v_c^2 = -\frac{4m^2(T_c)}{\lambda} = -\frac{2\lambda v^4}{c_6 m_h^2}$$

$$c_6 = \frac{2}{3}\frac{1}{1-\frac{2}{3}\frac{v_c^2}{v^2}}$$

$$v_c^2 < v^2 \ \ \& \ \lambda < 0 \Rightarrow$$

$$\frac{2}{3} < c_6 < 2$$

Either weak or strong

✓ However, already alarming from EFT point of view

→ EFT description might not make sense

# 1st order phase transition in EFT approach

## Case II: include all higher-dim terms with universal Wilson coefficients

:can be resumed up to infinite order in Higgs field and EFT can make sense as long as $E < \Lambda_{cutoff}$

$$\mathrm{V}_{EFT} = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\, v^2} \left(\frac{h^2}{2}\right)^{2+n}$$

NDA in SILH basis: $c_{4+2n} \sim \left(\frac{v}{f}\right)^{2n} \longrightarrow c_{4+2n} \sim c \left(\frac{v}{f}\right)^{2n} \equiv c\xi^n$

At tree-level

$$\lambda_3 \equiv \frac{d^3 V_{EFT}}{dh^3}\bigg|_{h=v} = \frac{3m_h^2}{v}\left[1 + 16c\, \frac{\xi}{(2-\xi)^4}\right]$$

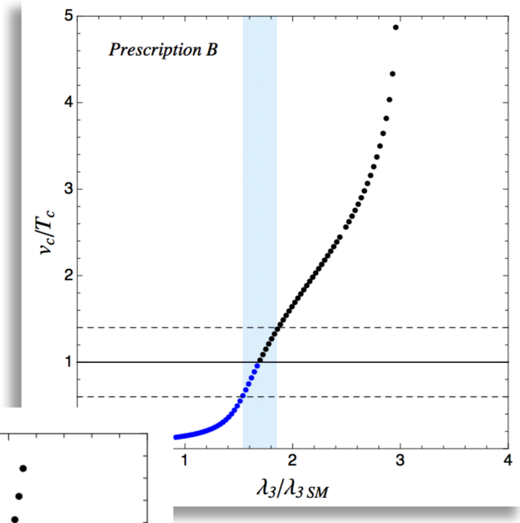
$$\lambda_4 \equiv \frac{d^4 V_{EFT}}{dh^4}\bigg|_{h=v} = \frac{3m_h^2}{v^2}\left[1 + 32c \frac{(6+\xi)\xi}{(2-\xi)^5}\right]$$

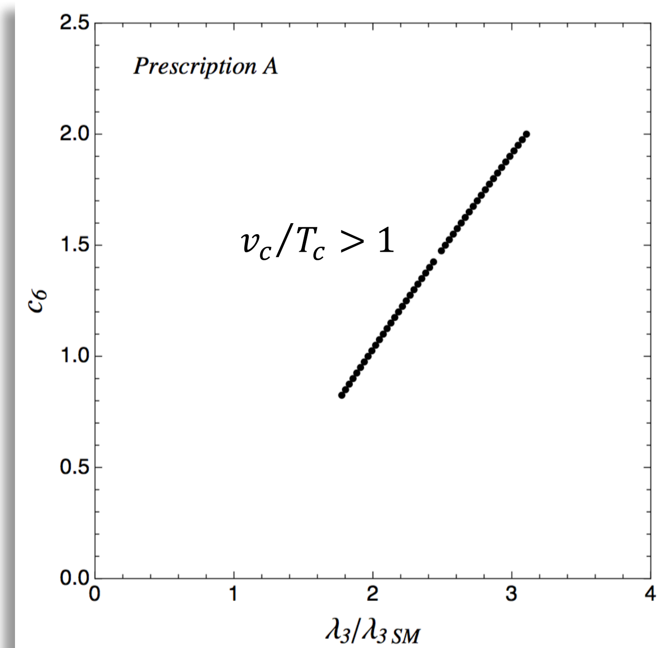
$$\frac{\lambda_4 / \lambda_4^{SM}}{\lambda_3 / \lambda_3^{SM}} = 2\, \frac{6+\xi}{2-\xi} \to \boxed{14} \text{ as } \xi \to 1 \text{ (or } f \to v)$$

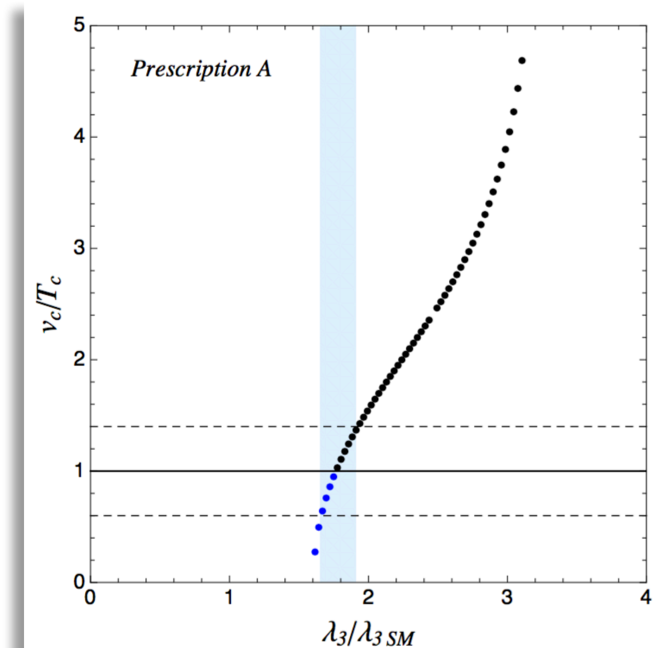
# 1st order phase transition in EFT approach

## Case I:  dim-6 operator

$$V_{EFT} = \frac{1}{2} m^2 h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{8}\frac{c_6}{v^2}\frac{m_h^2}{2\,v^2} h^6$$




$c_6 \sim \mathcal{O}(1)$ : expansion in terms of $H$ might not be justified. EFT not valid

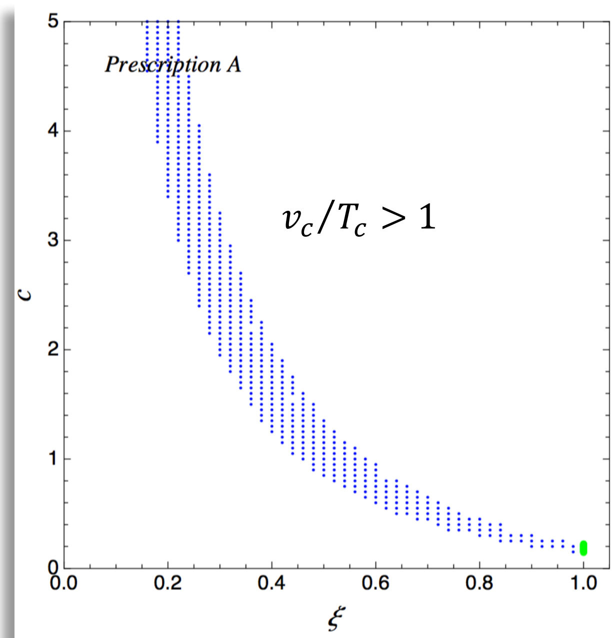
High-T approximation seems to be ok, e.g. no large mass involved

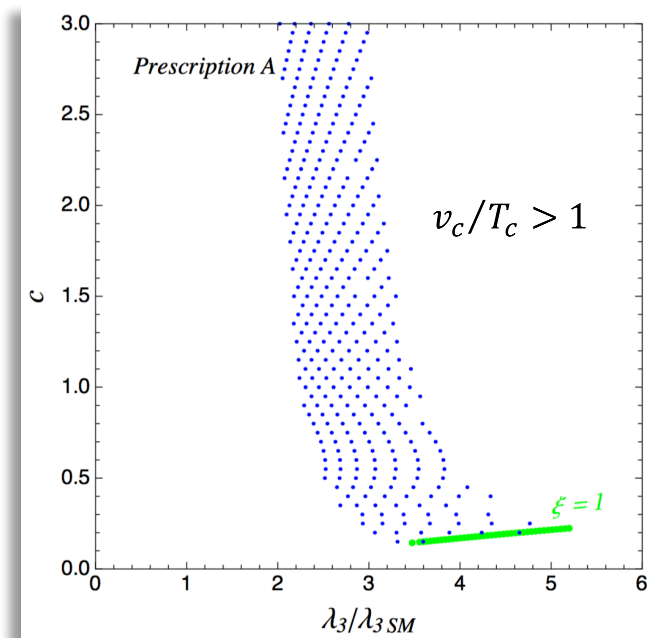
The uncertainty due to the finite $v_c/T_c$ is not pronounced

## Case II: all orders of $|H|^{4+2n}$

$$V_{EFT} = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\, v^2} \left(\frac{h^2}{2}\right)^{2+n}$$

where $c_{4+2n} \sim c \left(\frac{v}{f}\right)^{2n} \equiv c\xi^n$

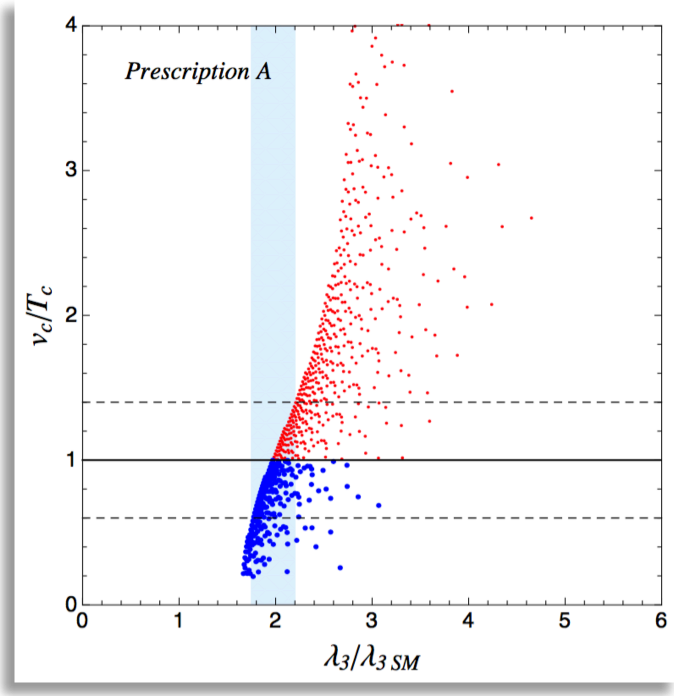


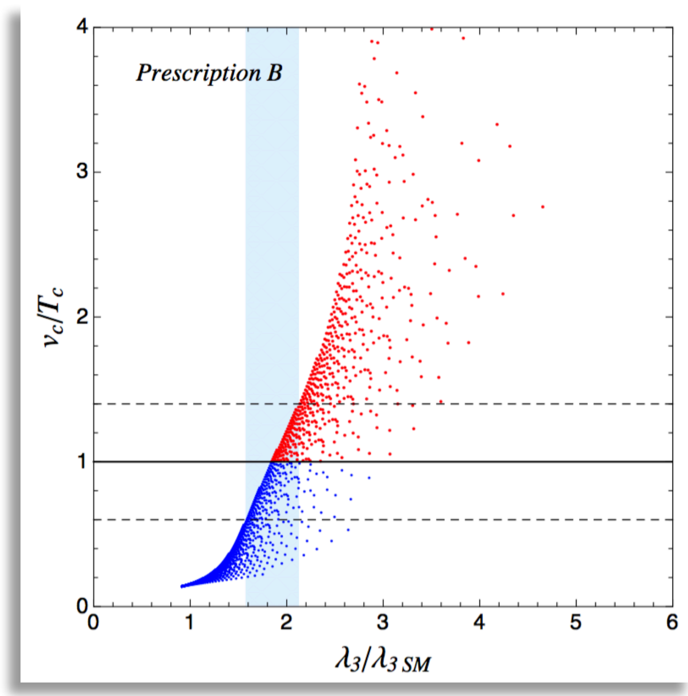



## Case II: all orders of $|H|^{4+2n}$

$$V_{EFT} = \frac{1}{2}m^2 h^2 + \frac{1}{4}\lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\,v^2}\left(\frac{h^2}{2}\right)^{2+n}$$

where $c_{4+2n} \sim c\left(\frac{v}{f}\right)^{2n} \equiv c\xi^n$




Similarly

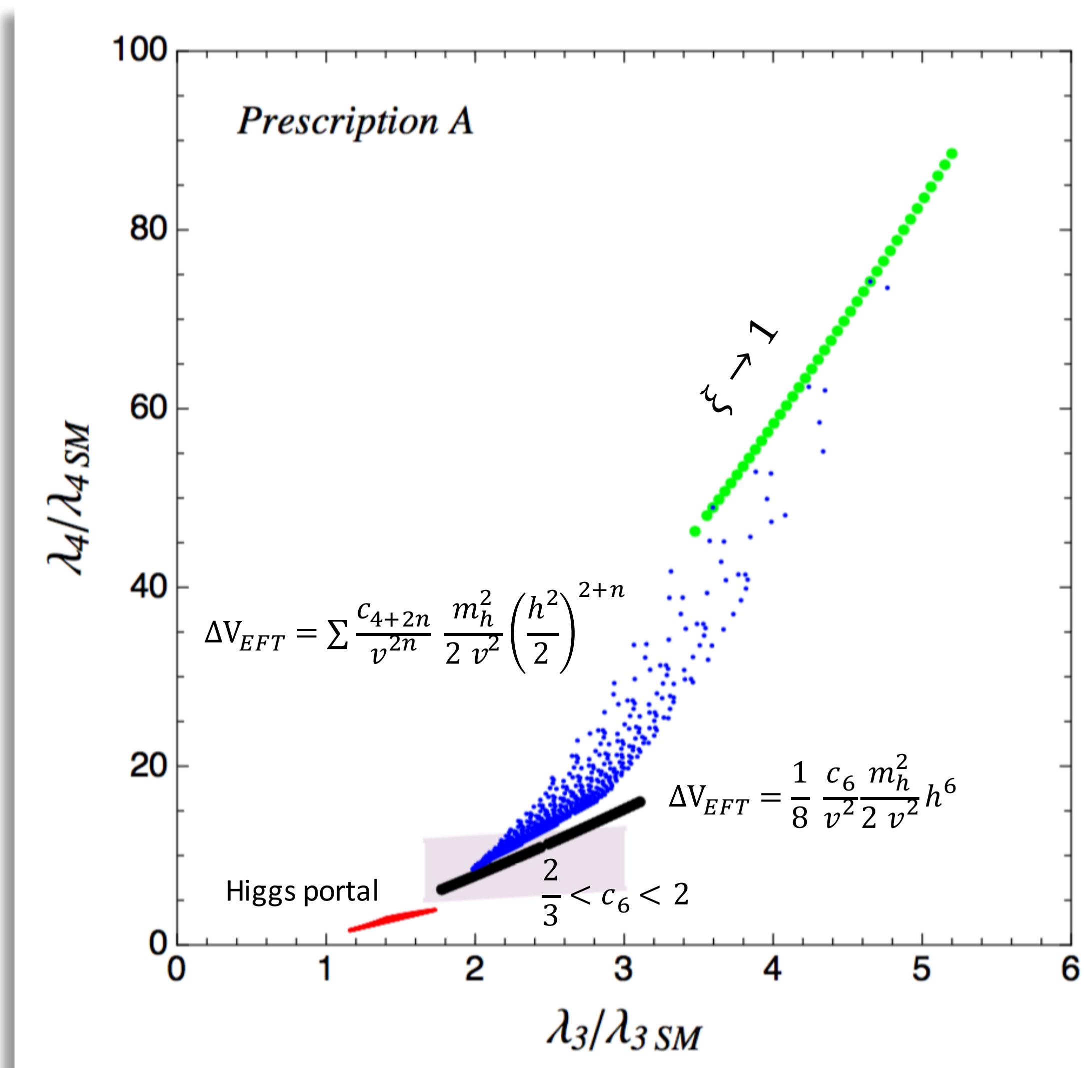
High-T approximation seems to be ok, e.g. no large mass involved

The uncertainty due to the finite $v_c/T_c$ is not pronounced

Strong 1st order
Electroweak Phase Transition

# Cubic vs. Quartic
# Higgs self-coupling

# Cubic vs. Quartic



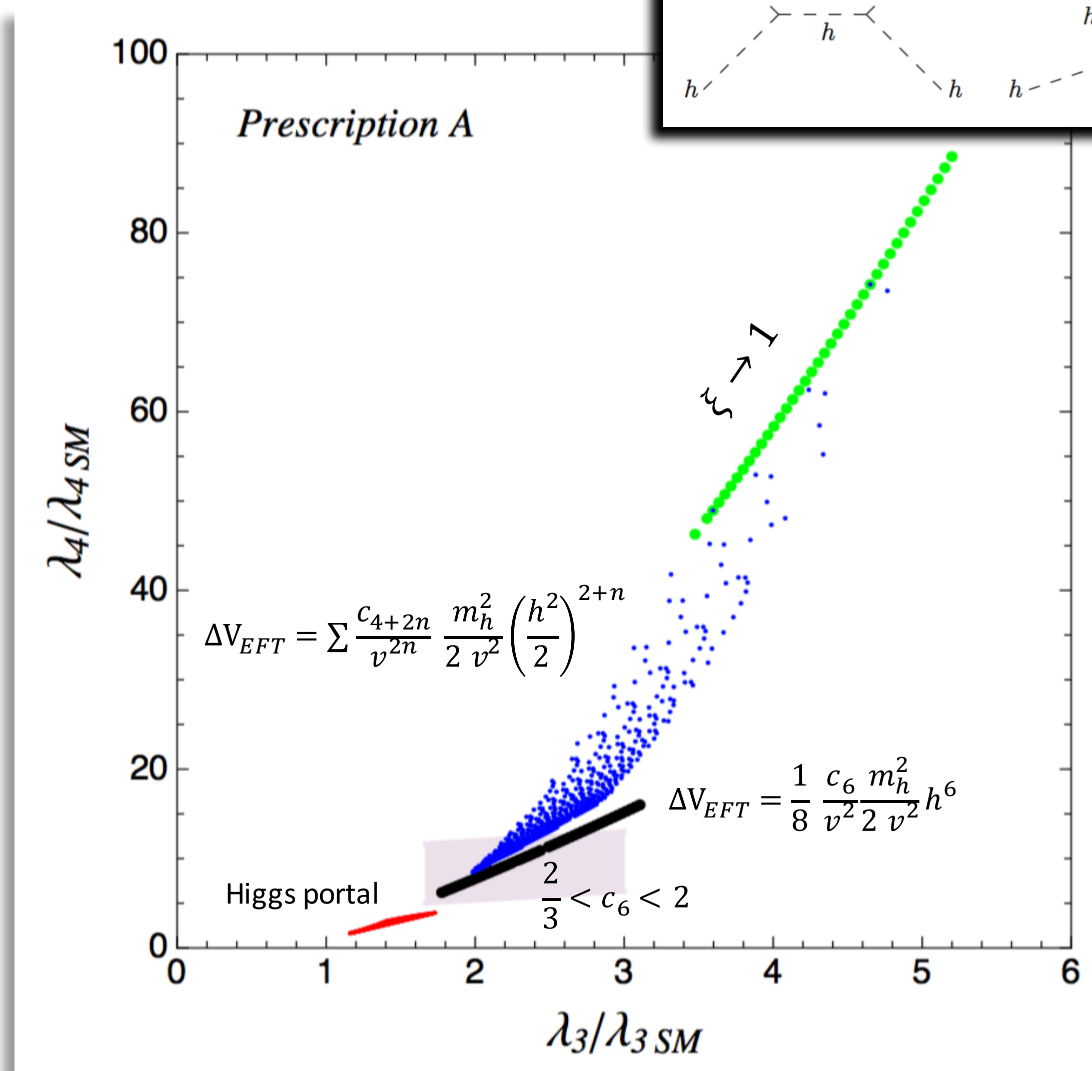

EFT prefers large Higgs self-couplings

Large quartic coupling might be able to be tested at future collider

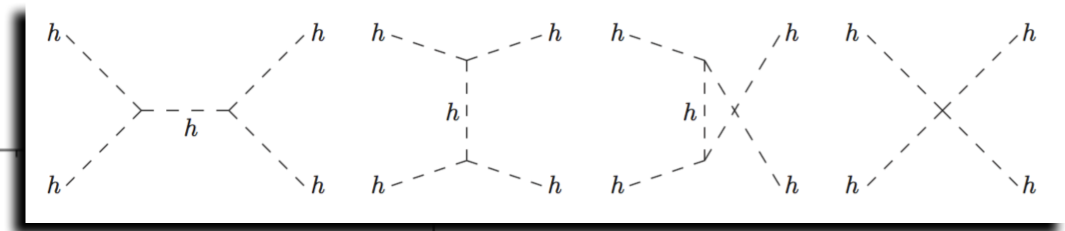
Unitarity bound

Electroweak Precision Test

# Cubic vs. Quartic



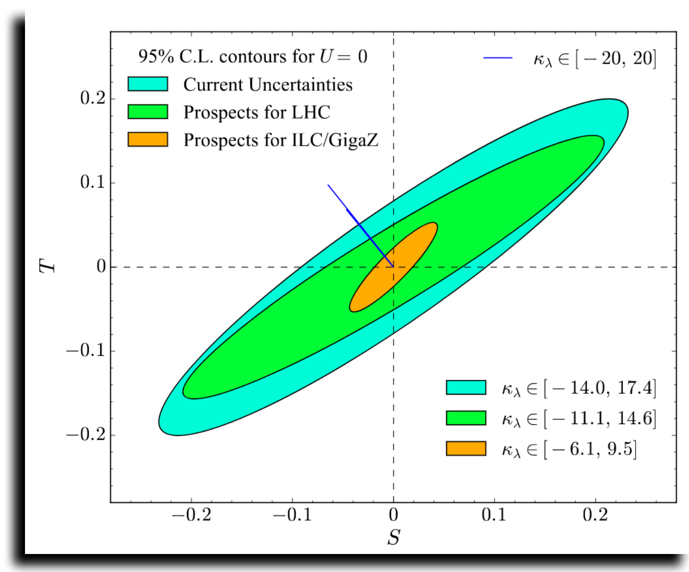

Unitarity bound



$$\left|\lambda_3 / \lambda_3^{SM}\right| \lesssim 6.5 \qquad \left|\lambda_4 / \lambda_4^{SM}\right| \lesssim 65$$

Luzio, Grober, Spannowsky 17’

Electroweak Precision Test

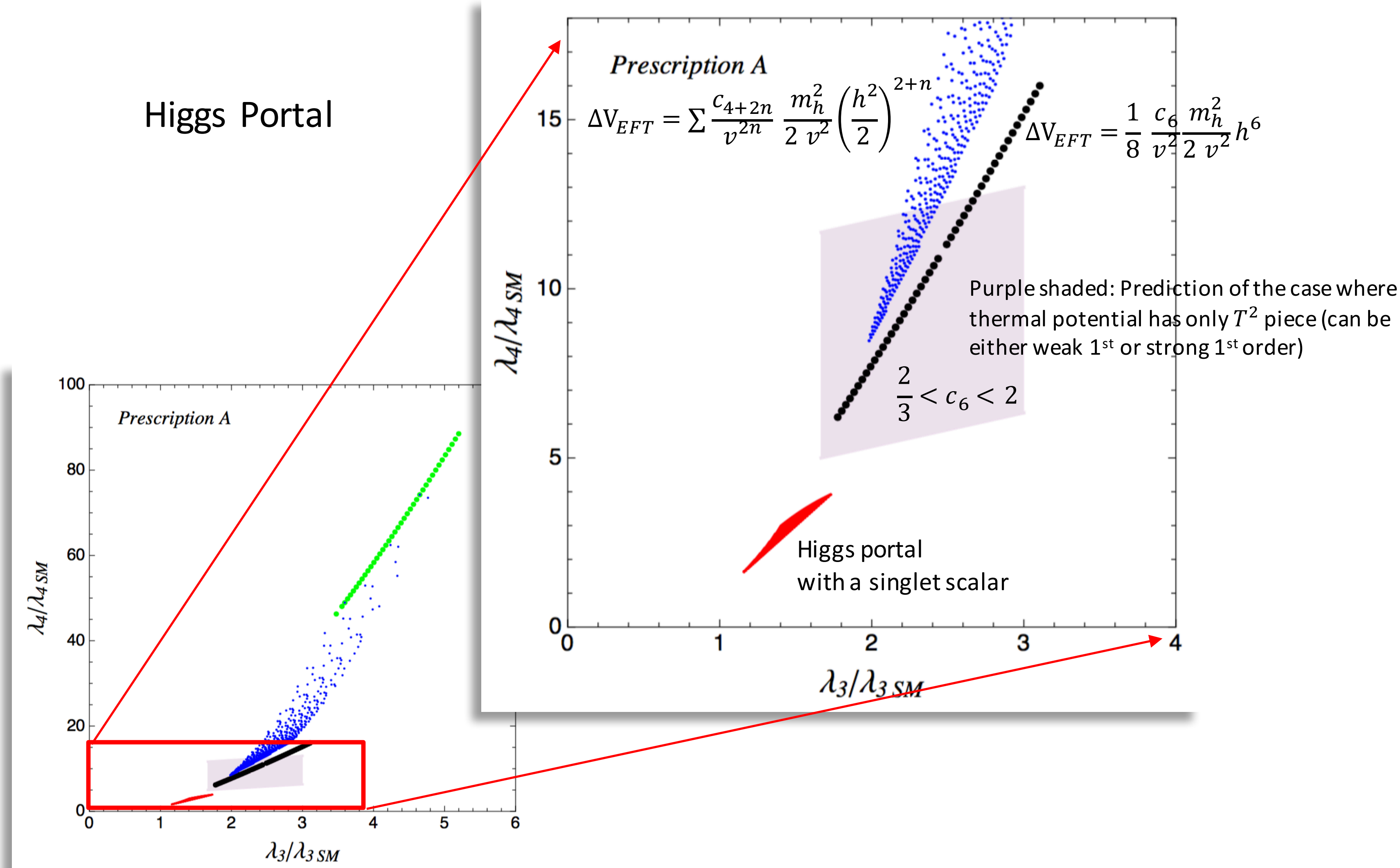



Kribs, Maier,Rzehak, Spannowsky, Waite 17’

# Cubic vs. Quartic

$$V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6}\left(\frac{3\, m_h^2}{v}\right) h^3 + d_4 \frac{1}{24}\left(\frac{3\, m_h^2}{v^2}\right) h^4$$

Higgs Portal



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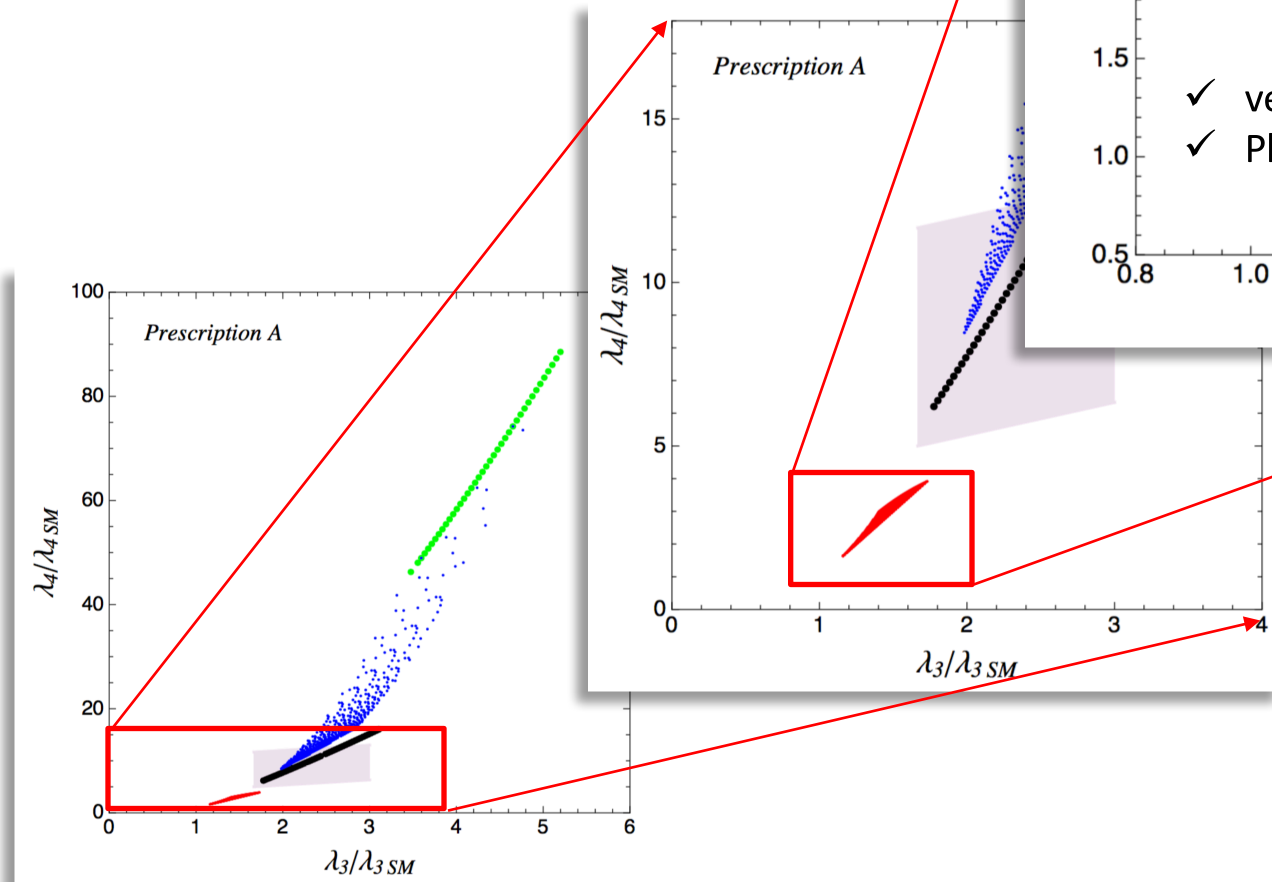
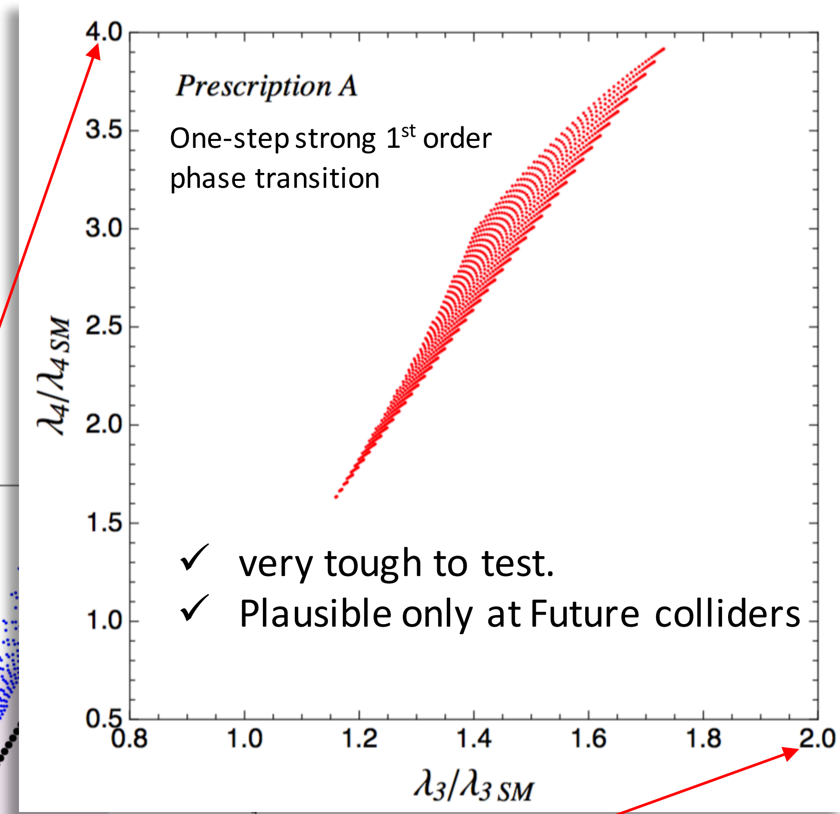
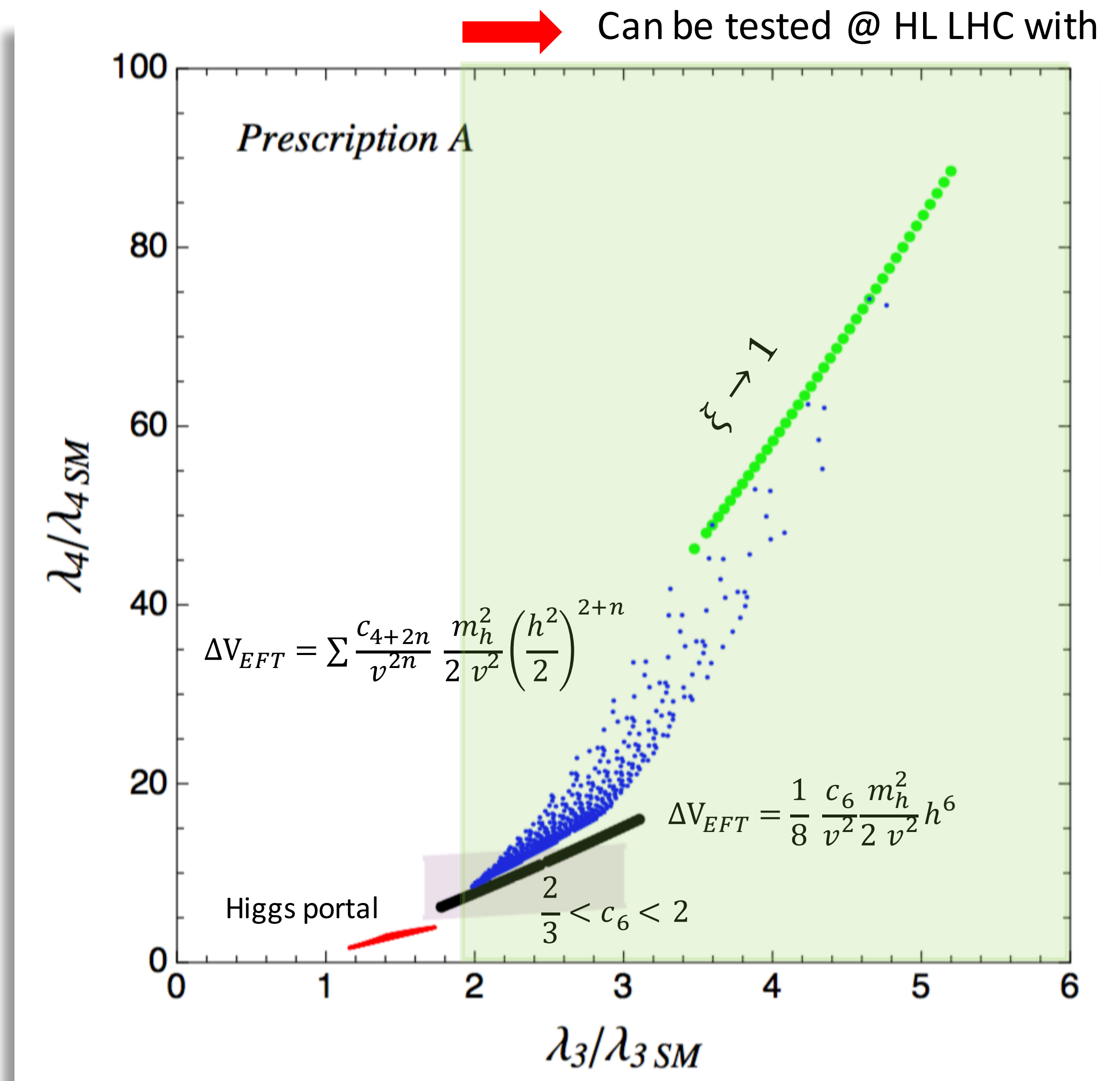
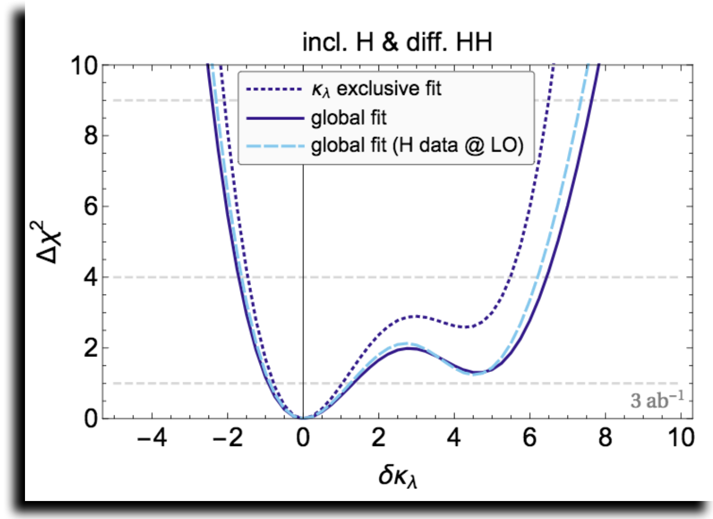
Higgs Portal



$\mathcal{O}(1)$ fraction of EFT parameter space can be tested at the HL LHC



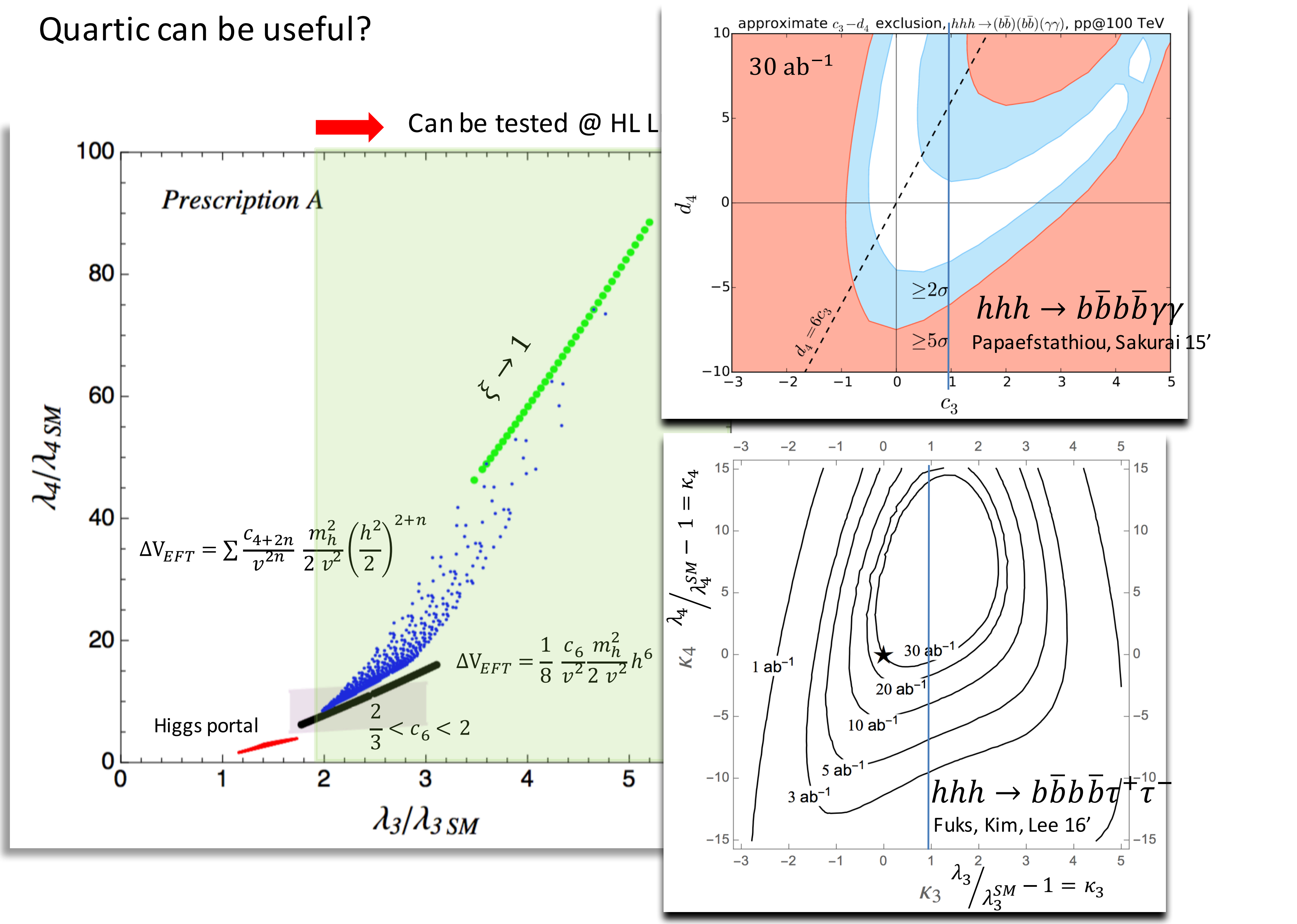




Vita, Grojean, Panico, Riembau, Vantalon 17’

$$c_3 = [0.1, 2.3]\ @1\sigma$$

Kim, Sakaki, SON in progress

# Quartic can be useful?



# Summary

- High-T approximation, finite v/T, A large coupling
  - ❑ Higgs Portal with a singlet scalar with Z2

    Do not use high-T approximation
    $\mathcal{O}(1)$ fluctuation on the precision of Higgs self-coupling due to v/T criteria
    → dramatic impact on future collider plan
    Resummation of thermal diagrams has to be done

  - ❑ Effective Field Theory Approach

    Above issues are mild
    Large deviation of coupling > Validity of EFT

- BSM maps in $(\lambda_3, \lambda_4)$

  $\lambda_4$ could be important discriminator of different scenarios
  It has chance @ 100 TeV in a situation that a large $\delta\lambda_3$ is observed

- Extra questions

  What if we see any hint for strong 1st order PT via HHH coupling?
  What can we do for non-perturbative region