# Higgs stabilization during inflation

Jinn-Ouk Gong

APCTP, Pohang 37673, Korea

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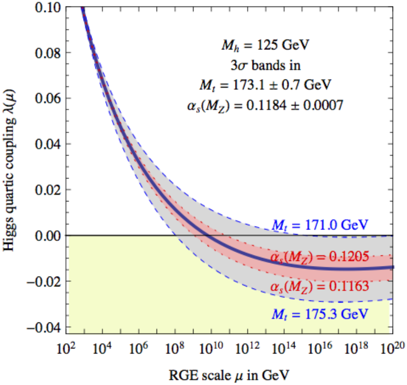
# Outline

# Unstable standard model vacuum

Higgs quartic coupling $\lambda_h$ turns negative at high energy scale

$$\frac{d\lambda_h}{d\log\mu} = \frac{1}{16\pi^2}\left(\cdots - 6y_t^4 + \cdots\right)$$

Instability develops at $\Lambda_{\max} \sim 10^{10-11}$ GeV (Degrassi et al. 2012)

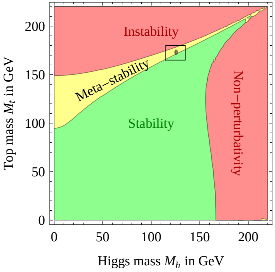


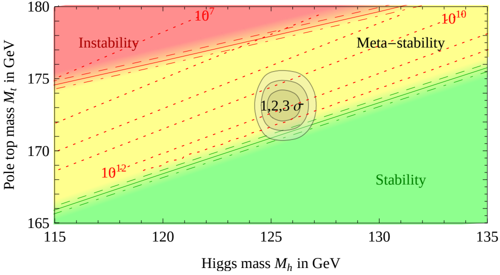
# Metastable EW vacuum

Not absolutely stable though, our EW vacuum is safe: given that

$$m_h = 125.09 \pm 0.21 \pm 0.11\text{GeV}$$
$$m_t = 173.34 \pm 0.76 \pm 0.3\text{GeV}$$

Lifetime of the EW vacuum $\gg$ age of the universe



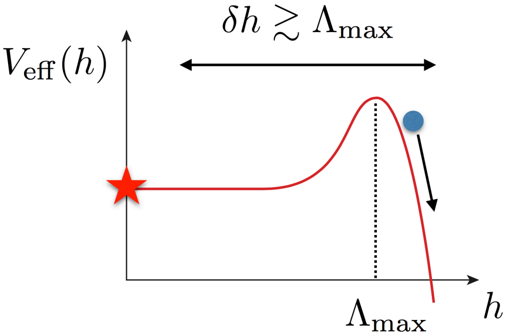

(Elias-Miro et al. 2012; Degrassi et al. 2012)

# Higgs stability during inflation

Hubble parameter during inflation may well be large:

$$H_{\rm inf} \lesssim A_{\mathcal{R}} \pi \sqrt{\frac{r}{2}} m_{\rm Pl} \approx 6.73 \times 10^{13} \left(\frac{r}{0.07}\right)^{1/2} \text{ GeV}$$

Higgs acquires quantum fluctuations $\delta h = \dfrac{H_{\rm inf}}{2\pi} \gtrsim \Lambda_{\rm max}$



# EW vacuum stabilization during inflation

- Low-scale inflation: $H_{\text{inf}} \lesssim 10^{10-11}$ GeV ($E_{\text{inf}} \lesssim 10^{14-15}$ GeV)
- Non-minimal coupling to gravity (Espinosa et al. 2008; Herranen et al. 2014)
- Thermal effects (Fairbairn & Hogan 2014)
- ✓ Direct coupling to inflaton (Lebedev & Westphal 2013; Kobakhidze & Spencer-Smith 2013)
- ✓ Coupling to another scalar (Gonderinger et al. 2010; Khan & Rakshit 2014)

# Lagrangian

Simplest case of a renormalizable Higgs-inflaton coupling

$$V(\phi, h) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}\xi\phi^2 h^2$$

$$\text{with} \quad |\lambda_{\min}|\frac{m_\phi}{\phi} \sim 10^{-9} \leq \xi \leq 4\sqrt{2}\pi\frac{m_\phi}{\phi} \sim 10^{-6}$$

- Upper bound: small radiative corrections
- Lower bound: cross term dominates

# Higgs mass during inflation

Cross term dominates (about $\lesssim 3.2$ $e$-folds) $\to$ SR inflation

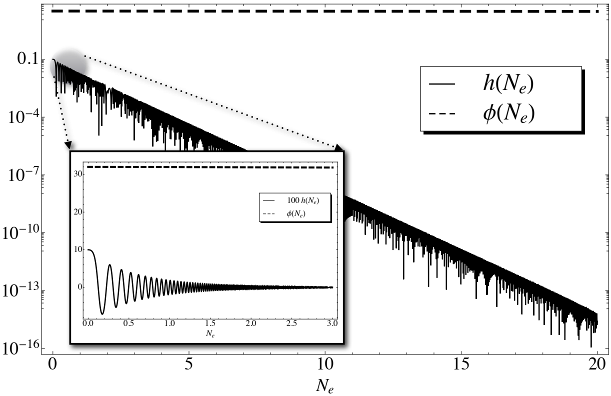
$$\frac{m_h}{H} \gtrsim \begin{cases} 2\sqrt{\dfrac{3|\lambda_{\min}|}{\xi}}\dfrac{m_{\rm Pl}}{\phi_{\rm eq}} \sim 18 & \text{for} \quad h_{\rm eq} \lesssim h \lesssim h_{\max} \\ \sqrt{6\xi}\dfrac{m_{\rm Pl}}{m_\phi} \sim 76 & \text{for} \quad h \lesssim h_{\rm eq} \end{cases}$$

Being very heavy, Higgs is oscillating with the amplitude decreasing

$$h = h_{\rm eq}\left(\frac{a_{\rm eq}}{a}\right)^{3/2}\cos\left(\mu H_{\rm inf} t\right) \sim a^{-3/2} \quad \text{where} \quad \mu \equiv \sqrt{\frac{\xi\phi_{\rm eq}^2}{H_{\rm inf}^2} - \frac{9}{4}} \approx \frac{m_h}{H_{\rm inf}}$$

# Evolution of the Higgs and the inflaton

$h \sim a^{-3/2}$ drops to EW scale after 20 - 25 $e$-folds (Lebedev and Westphal 2013)




N.B. $\dfrac{\Delta\phi}{\phi_{\rm ini}} \sim \left(\dfrac{3h_{\rm eq}}{2\phi_{\rm ini}}\right)^2 \ll 1$ so that $\phi \approx \phi_{\rm ini} \approx \phi_{\rm eq}$: $\phi$ keeps slow-roll

# Features of the heavy Higgs on the power spectrum

Higgs = effectively heavy field coupled to inflaton

1. From oscillating BG: sinusoidal Higgs sol contributes to $H$

$$\frac{\Delta\mathcal{P}_{\mathcal{R}}^{(\mathrm{bg})}}{\mathcal{P}_{\mathcal{R}}^{(0)}} = -\frac{3}{2}\frac{\mu}{\epsilon_0}\left(\frac{k_{\mathrm{eq}}}{k}\right)^3\left\{\cos\left[2\mu\log\left(\frac{k}{k_\star}\right)+2\mu N_\star+2\alpha_\star\right]+\frac{1}{\mu}\sin\left[2\mu\log\left(\frac{k}{k_\star}\right)+2\mu N_\star+2\alpha_\star\right]\right\}$$

2. From quantum fluctuations: using the in-in formalism

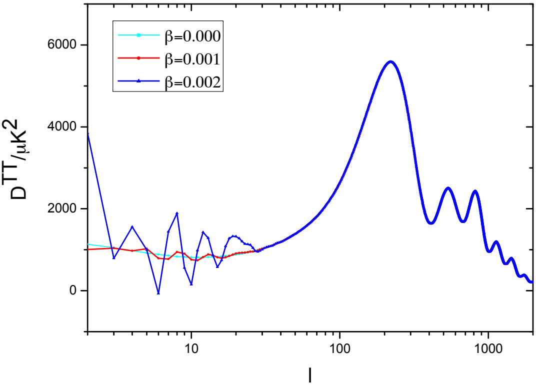
$$\frac{\Delta\mathcal{P}_{\mathcal{R}}^{(\mathrm{q})}}{\mathcal{P}_{\mathcal{R}}^{(0)}} = \frac{3}{2}\mu\epsilon_0\left(\frac{k_\star}{k}\right)^3\left(\frac{\pi^2}{2}+\frac{1}{4\mu^2}\right)$$

Effects from oscillating BG dominate the corrections (JG, Han and Pi 2015)

# Trail of the Higgs in the power spectrum

With $k_{\text{eq}}$ being arbitrary, introducing a new parameter $\beta \equiv k_{\text{eq}}/k_\star$

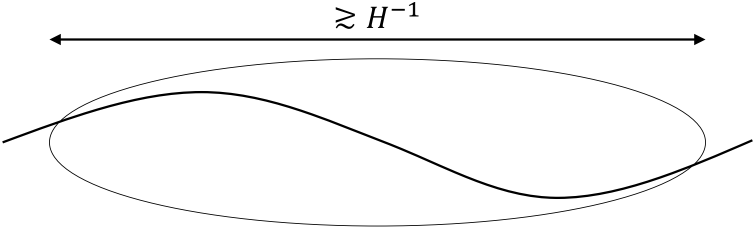
$$\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}^{(0)}} = -\frac{3}{2}\frac{\mu}{\epsilon_0}\left(\beta\frac{k_\star}{k}\right)^3 \cos\left\{2\mu\left[\log\left(\frac{k}{k_\star}\right)+N_k+\frac{2}{3}\sinh^{-1}1-\log\beta\right]\right\}+\cdots$$




(JG, Han and Pi 2015)

1. Introduction
2. Higgs-inflaton coupling
   - Background evolution
   - Heavy Higgs in the power spectrum
3. Stochastic stabilization
   - Stochastic formalism
   - Stochastic evolution of Higgs and singlet
   - Post-inflationary evolution
4. Conclusions

# Stochasicity on super-horizon scales




(Starobinsky 1986)

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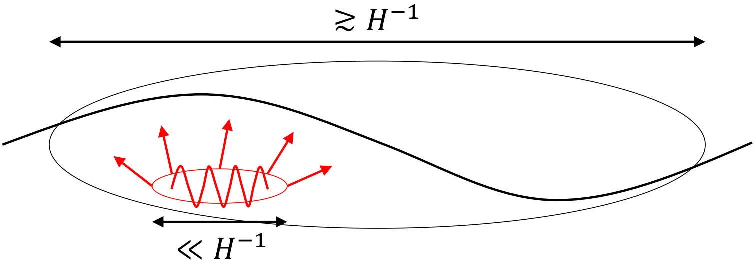


(Starobinsky 1986)

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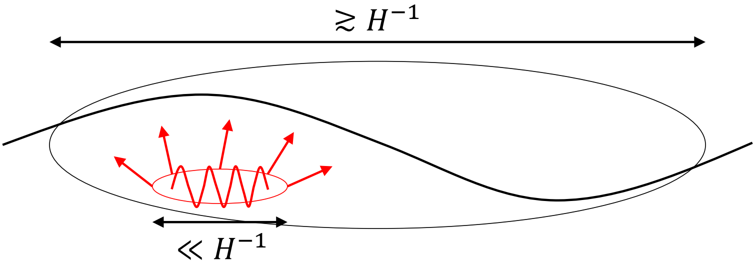


$$\phi(t,\boldsymbol{x}) = \varphi(t,\boldsymbol{x}) + \int \frac{d^3k}{(2\pi)^3} W(k-\epsilon aH)\left[a_{\boldsymbol{k}}\hat{\varphi}_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + c.c.\right]$$

(Starobinsky 1986)

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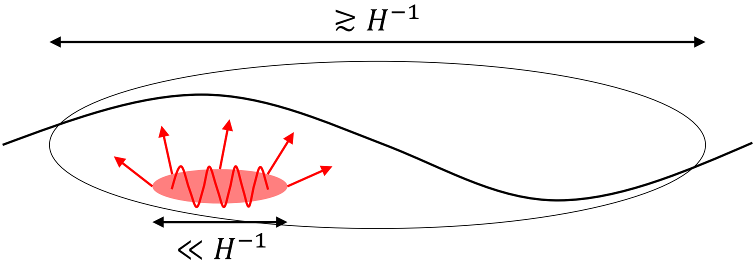
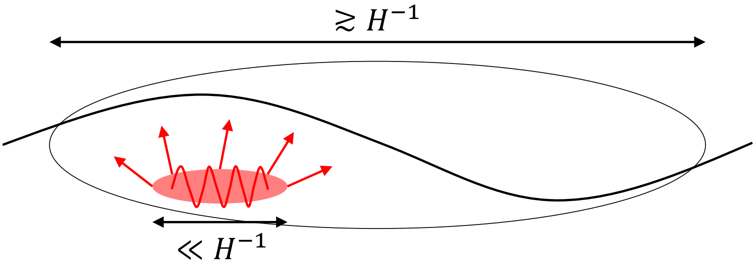



$$\phi(t,\boldsymbol{x}) = \varphi(t,\boldsymbol{x}) + \int \frac{d^3k}{(2\pi)^3} \underbrace{W(k-\epsilon aH)}_{\text{Window function that vanishes if } \lambda \gtrsim (\varepsilon H)^{-1}} \left[a_{\boldsymbol{k}} \hat{\varphi}_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + c.c.\right]$$

(Starobinsky 1986)

# Stochasicity on super-horizon scales



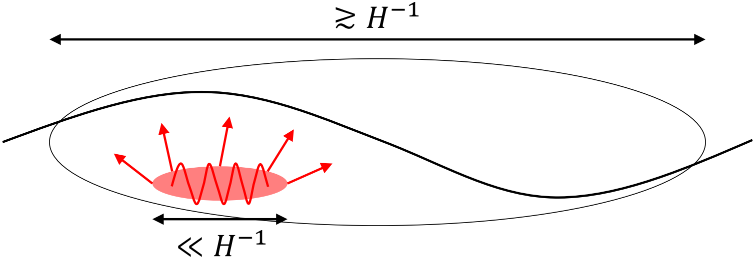

$$\phi(t,\boldsymbol{x}) = \varphi(t,\boldsymbol{x}) + \int \frac{d^3k}{(2\pi)^3} W(k-\epsilon aH)\left[a_{\boldsymbol{k}}\hat{\varphi}_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + c.c.\right]$$

"Coarse-grained" field with long-wavelength modes ($\gtrsim H^{-1}$) only

Short-wavelength modes only because of the window function: random "noise" $\xi_\varphi$

(Starobinsky 1986)

# Stochasicity on super-horizon scales




Langevin equation: $$\frac{d\varphi}{dN} + \frac{V'}{3H^2} = \underbrace{\frac{H}{2\pi}\xi_\varphi(N)}$$

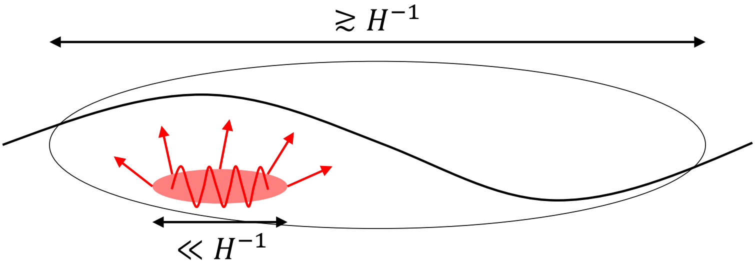
Classical drift

Gaussian noise satisfying

$$\langle \xi_\varphi(N)\xi_\varphi(N') \rangle = \delta(N - N')$$

(Starobinsky 1986)

# Stochasicity on super-horizon scales




Langevin equation: $\dfrac{d\varphi}{dN} + \dfrac{V'}{3H^2} = \dfrac{H}{2\pi}\xi_\varphi(N)$

Indeed, for free field ($V = 0$)

$$\varphi = \frac{1}{2\pi}\int_{N_i}^{N} dN'\, H\xi_\varphi \to \langle\varphi^2(N)\rangle = \left(\frac{H}{2\pi}\right)^2 (N - N_i)$$

(Starobinsky 1986)

# Evolution of probability

Higgs is coupled to a light singlet scalar $S$

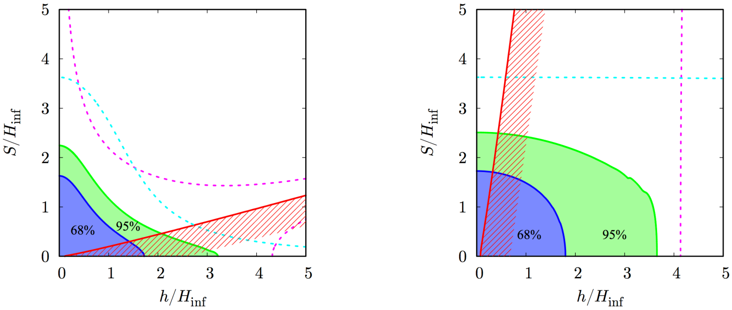
$$V(h,S)=\frac{1}{4}\lambda h^4+\frac{1}{2}m_S^2S^2+\frac{1}{4}\lambda_S S^4+\frac{1}{2}\lambda_{hS}h^2S^2$$

Evolution of the probability distribution by Fokker-Planck equation

$$\frac{\partial P(h,S,N)}{\partial N}=\sum_{\phi=h,S}\frac{\partial}{\partial\phi}\left[\frac{V_\phi P(h,S,N)}{3H^2}+\frac{H^2}{8\pi^2}\frac{\partial P(h,S,N)}{\partial\phi}\right]$$

1. $P(h,S,N=0)=\delta(h)\delta(S)$
2. $P(\Lambda_h,S,N)=P(h,\Lambda_S,N)=0$ where $|V_h|_{h=\Lambda_h}=|V_S|_{S=\Lambda_S}=\dfrac{3H_{\rm inf}^3}{2\pi}$

# Modified distribution




(a) $\lambda_{hS} = 0.1$ (b) $\lambda_{hS} = 10^{-4}$

$$H_{\rm inf} = 10^{11}\ \text{GeV}, \quad N = 60, \quad \lambda_S = 0.01$$

In the presence of interaction, distribution is along the axes

(JG & Kitajima 2017)

# Analytic solution for probability

Assume $P(h,S,N) = \Psi(N)\Phi(h,S)$ with $\Psi \propto e^{-\alpha N}$ (Espinosa et al. 2008)

$$P(h,S,N) = \frac{1}{\Lambda_h\Lambda_S}\sum_{n,m\geq 0} e^{-\alpha_{nm}N}\cos\left[\pi\left(n+\frac{1}{2}\right)\frac{h}{\Lambda_h}\right]\cos\left[\pi\left(m+\frac{1}{2}\right)\frac{S}{\Lambda_S}\right]$$

where $\alpha_{nm} = \left(n+\frac{1}{2}\right)^2\frac{H_{\rm inf}^2}{8\Lambda_h^2} + \left(m+\frac{1}{2}\right)^2\frac{H_{\rm inf}^2}{8\Lambda_S^2}$

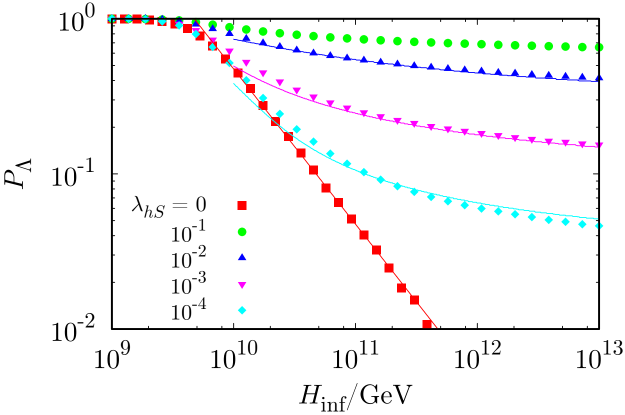
# Survival probability

The probability for our metastable universe (Espinosa et al. 2008; Hook et al. 2015)

$$
\begin{aligned}
P_\Lambda &= \int_{-\Lambda_S}^{\Lambda_S} dS \int_{-\Lambda_{\max}}^{\Lambda_{\max}} dh P(h,S) \\
&= \frac{4p}{\pi^2} \sum_{n,m\geq 0} e^{-q\alpha_{nm}N} \frac{-\dfrac{m+1/2}{n+1/2}\Lambda_h^2 \sin\left[\beta\pi\left(n+\dfrac{1}{2}\right)\dfrac{\Lambda_S}{\Lambda_h}\right] + \beta\Lambda_h\Lambda_S}{\beta^2(n+1/2)^2\Lambda_S^2 - (m+1/2)^2\Lambda_h^2}
\end{aligned}
$$

where $\beta \equiv \Lambda_{\max}/S$ and $p$ and $q$ are numerical fudge factors of $\mathcal{O}(1)$

# Enhancement of survival probability




$N = 60\,, \quad \lambda_S = 0.01 \quad$ (analytic curves for $p = 2$ and $q = 5$)

(JG and Kitajima 2017)

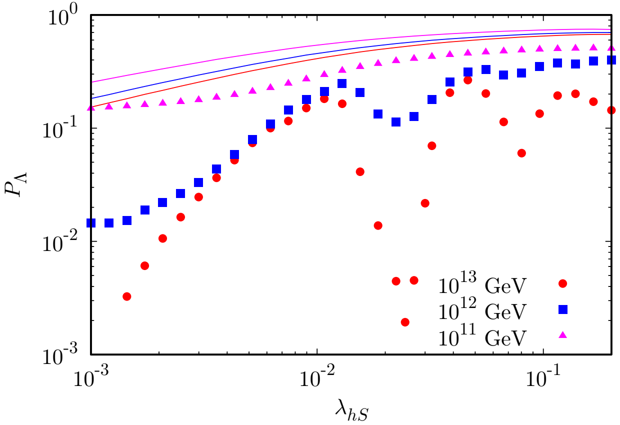
# Post-inflationary evolution

$$\ddot{h}+3H\dot{h}-\frac{\Delta}{a^2}h+\left(\lambda+\frac{1}{4}\frac{\partial\lambda}{\partial\log h}\right)h^3+\lambda_{hS}S^2h=0$$

$$\ddot{S}+3H\dot{S}-\frac{\Delta}{a^2}S+\lambda_S S^3+\lambda_{hS}h^2S=0$$

Since $m_S \ll H_{\rm inf}$, parametric resonance is efficient (Greene et al. 1997) leading to destabilization of EW vacuum (Ema et al. 2016)

# Survival probability with parametric resonance



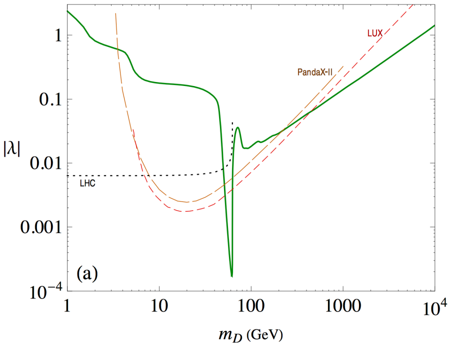

$$N = 60\,, \quad \lambda_S = 0.01$$

Parametric resonance after inflation does not destroy the chance

(JG and Kitajima 2017)

# Light singlet as dark matter

Light scalar $S$ may play the role of Higgs-portal DM




$m_S \gtrsim 500$ GeV for $\lambda_{hS} \gtrsim 0.1$ or $m_S \sim m_h/2$ for $\lambda_{hS} \sim 10^{-3} - 0.01$

(He and Tandeam 2016)

# Conclusions

- EW vacuum on the edge during inflation
- Direct coupling to the inflaton
  - Higgs becomes very massive and driven to EW vacuum
  - May leave oscillating features in the power spectrum
- Stochastic stabilization by a light scalar
  - Multi-field Fokker-Planck equation
  - High chance of landing in the EW vacuum
  - Light scalar may play the role of DM