Higgs inflation (circa 2017)

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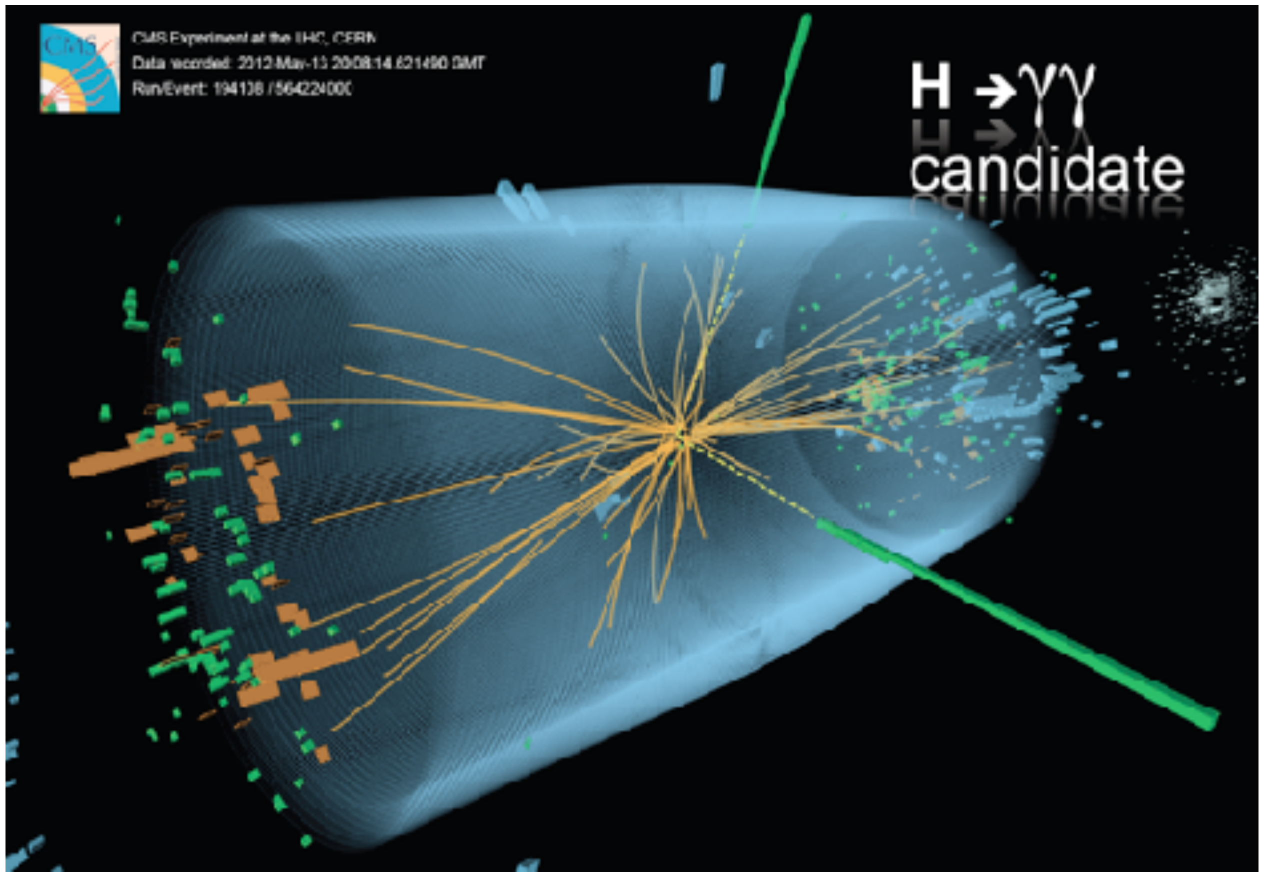
Higgs Mini-Workshop:
Precision, BSM and Cosmology
SNU, Sept 28, 2017.

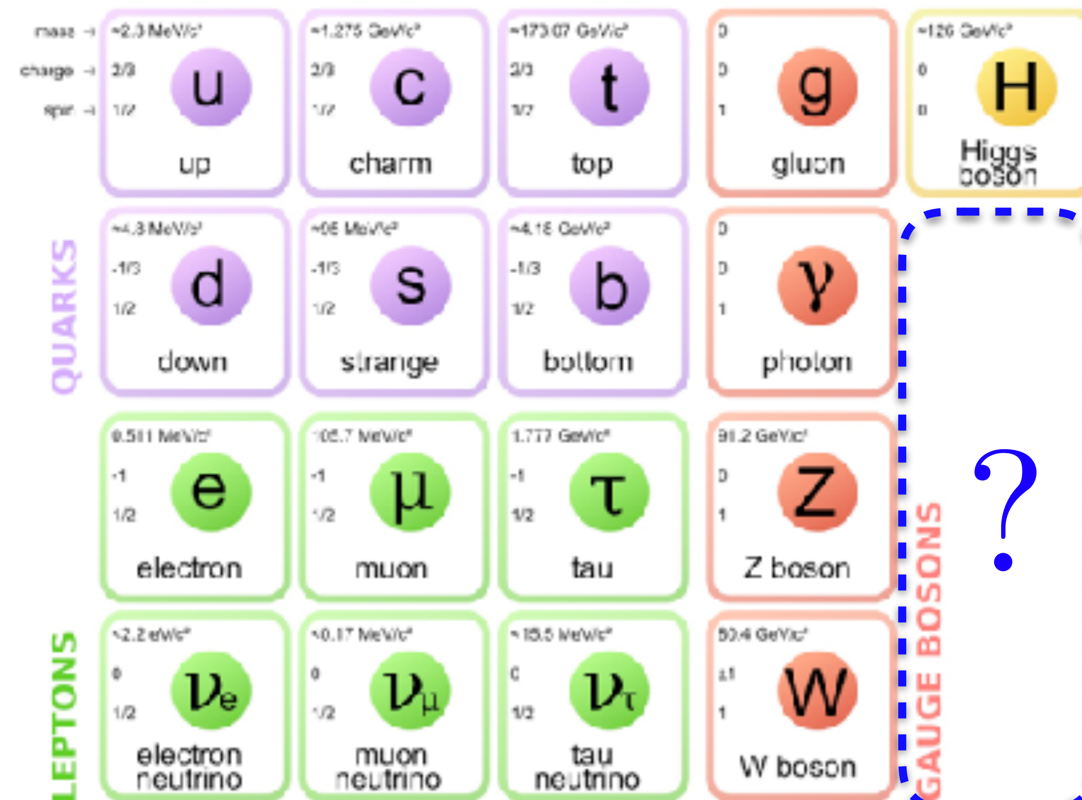
Outline

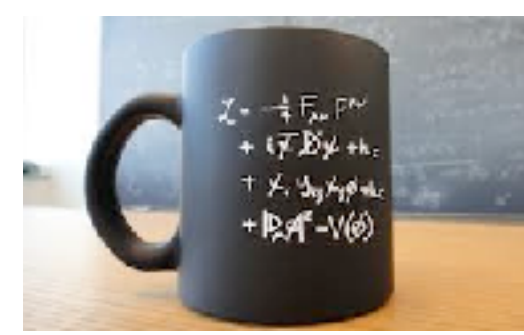
- Introduction
- Higgs inflation and validity
- UV complete Higgs inflation
- Conclusions

Standard Model

- Was the SM Higgs boson the last missing piece?





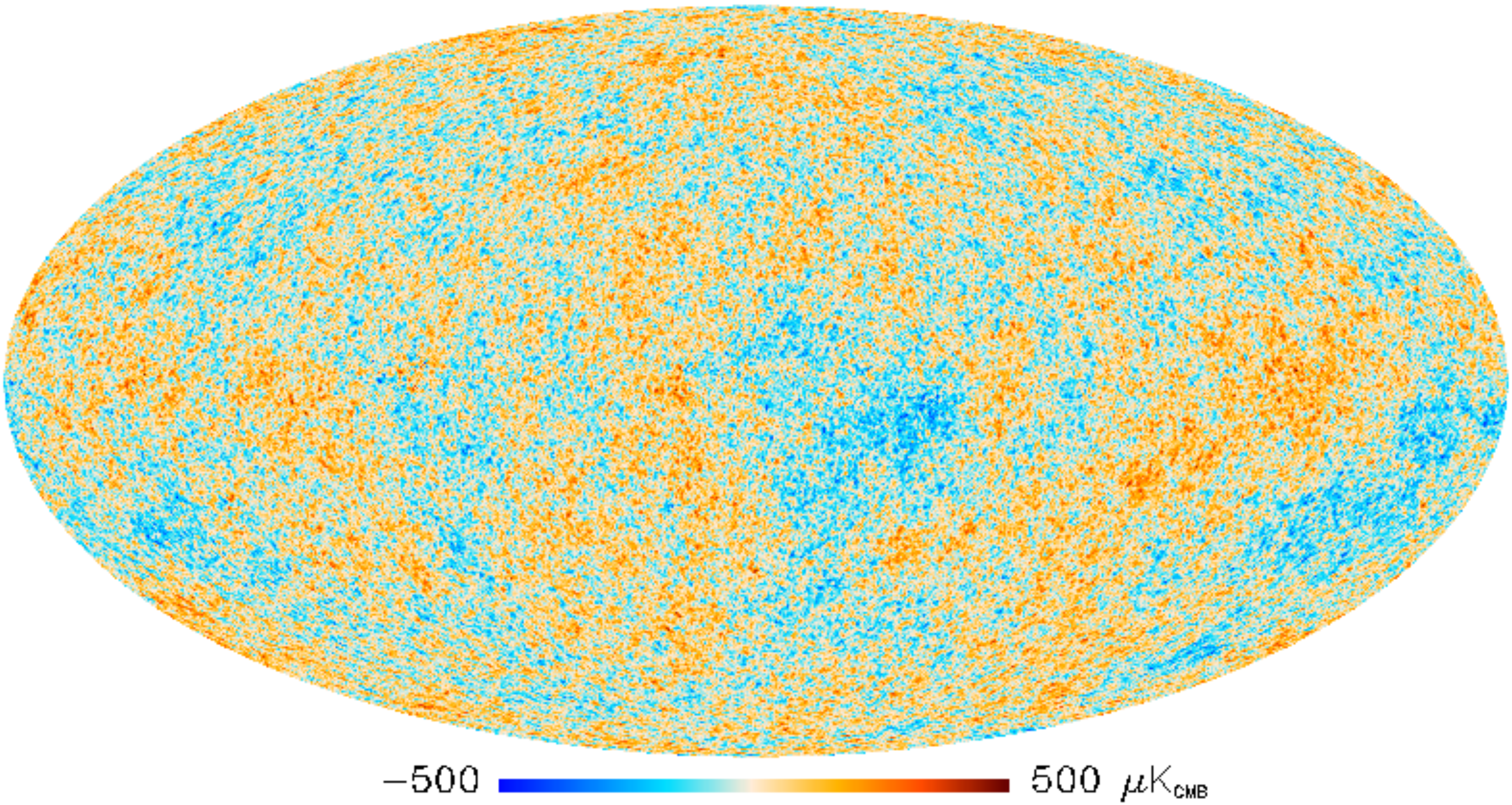


The Standard Model at renormalizable level has been established.

But, there are still lots of problems to be understood.

Inflation, hierarchy problem, flavor problem, dark matter, baryon asymmetry, dark energy, etc.

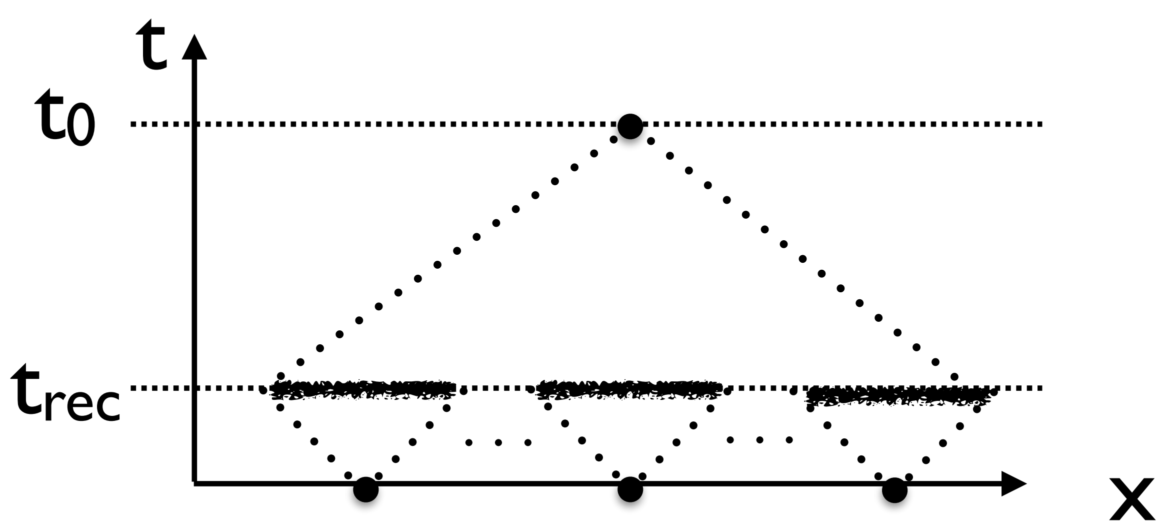
Cosmic Microwave Background

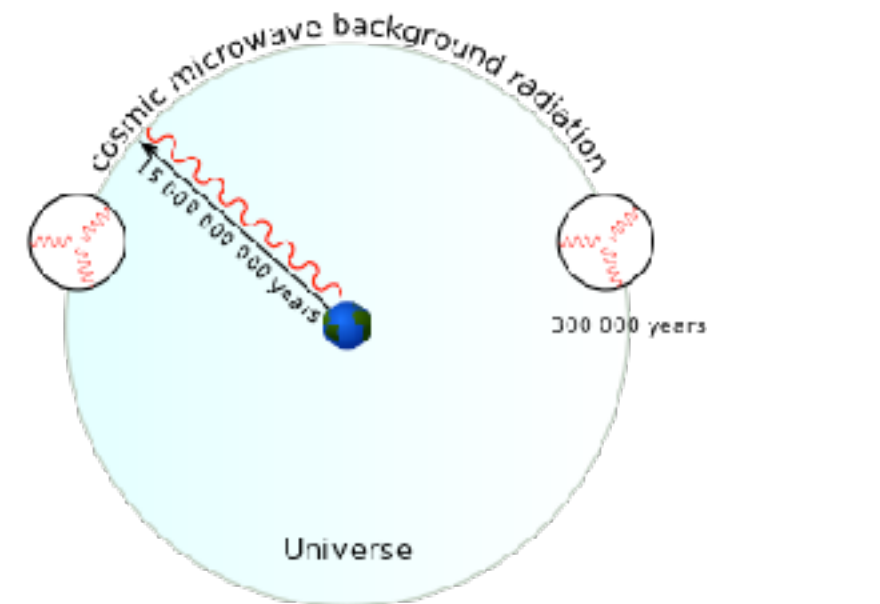


CMB temperature is homogeneous & isotropic by 1 in 10^4

Problems in cosmology

- Horizon problem: CMB once out of thermal contact?





L_{rec}: Proper distance at recombination.

$$L_{\rm rec} = a_{\rm rec}\int_0^{t_{\rm rec}} \frac{t}{a(t)} = \frac{1}{H_{\rm rec}} \quad \Rightarrow \quad L_{\rm rec} = \frac{1}{H_0}\left(\frac{1}{1+z_{\rm rec}}\right)^{3/2} \simeq \boxed{0.12\,{\rm Mpc.}}$$

$D_{rec,0}$: Distance to last scattering surface.

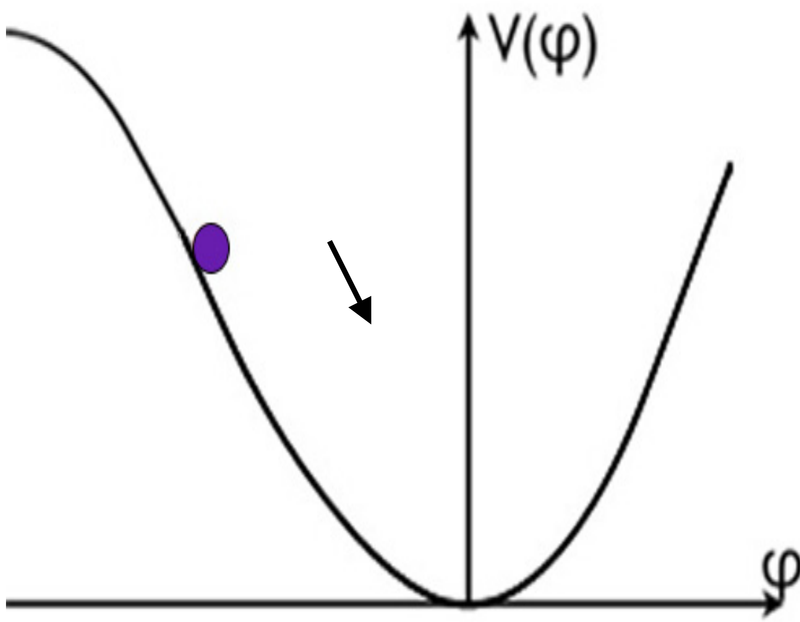
$$D_{\rm rec} = \frac{a_{\rm rec}}{a_0} D_0 = \frac{1}{1+z_{\rm rec}}\frac{2}{H_0} \simeq \boxed{8\,{\rm Mpc.}} \qquad ({\rm cf.}\ H_0^{-1} \simeq 4.4\,{\rm Gpc})$$

$\Rightarrow$ Causally connected region: $\theta = \dfrac{L_{\rm rec}}{D_{\rm rec}} \sim 1^{\rm o}.$

How uniform the CMB temperature 1 in 10^4?

Cosmic inflation

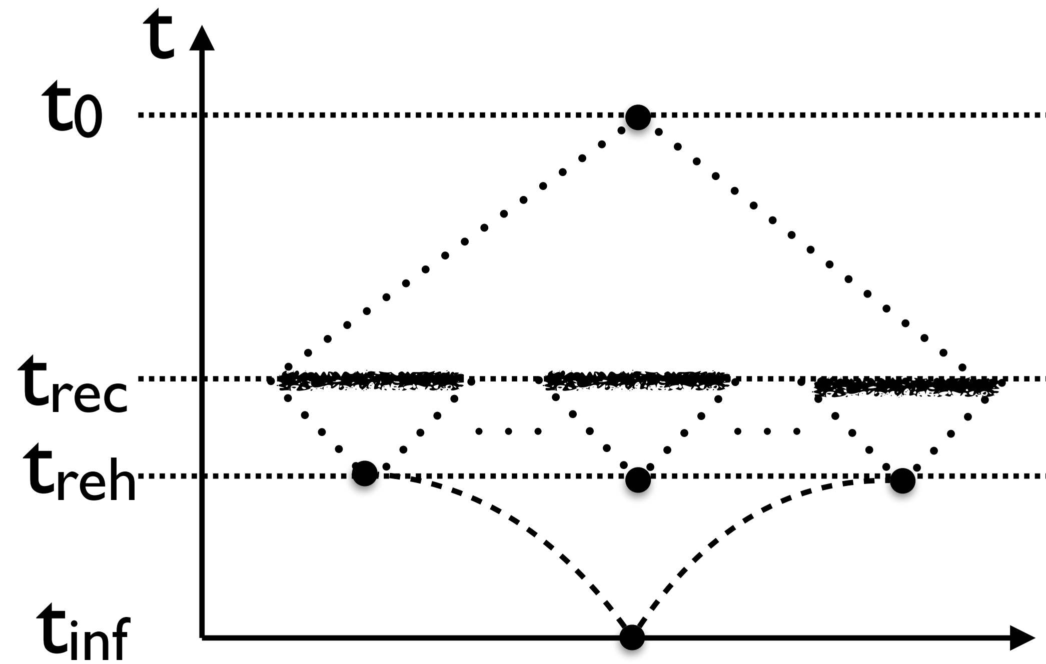
- Exponential expansion with a scalar field "inflaton" makes the CMB region once causally connected.



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$Ht \sim N \sim 50$$

"exponentially" large universe



- Explains homogeneity, flatness, relics, and structure formation.

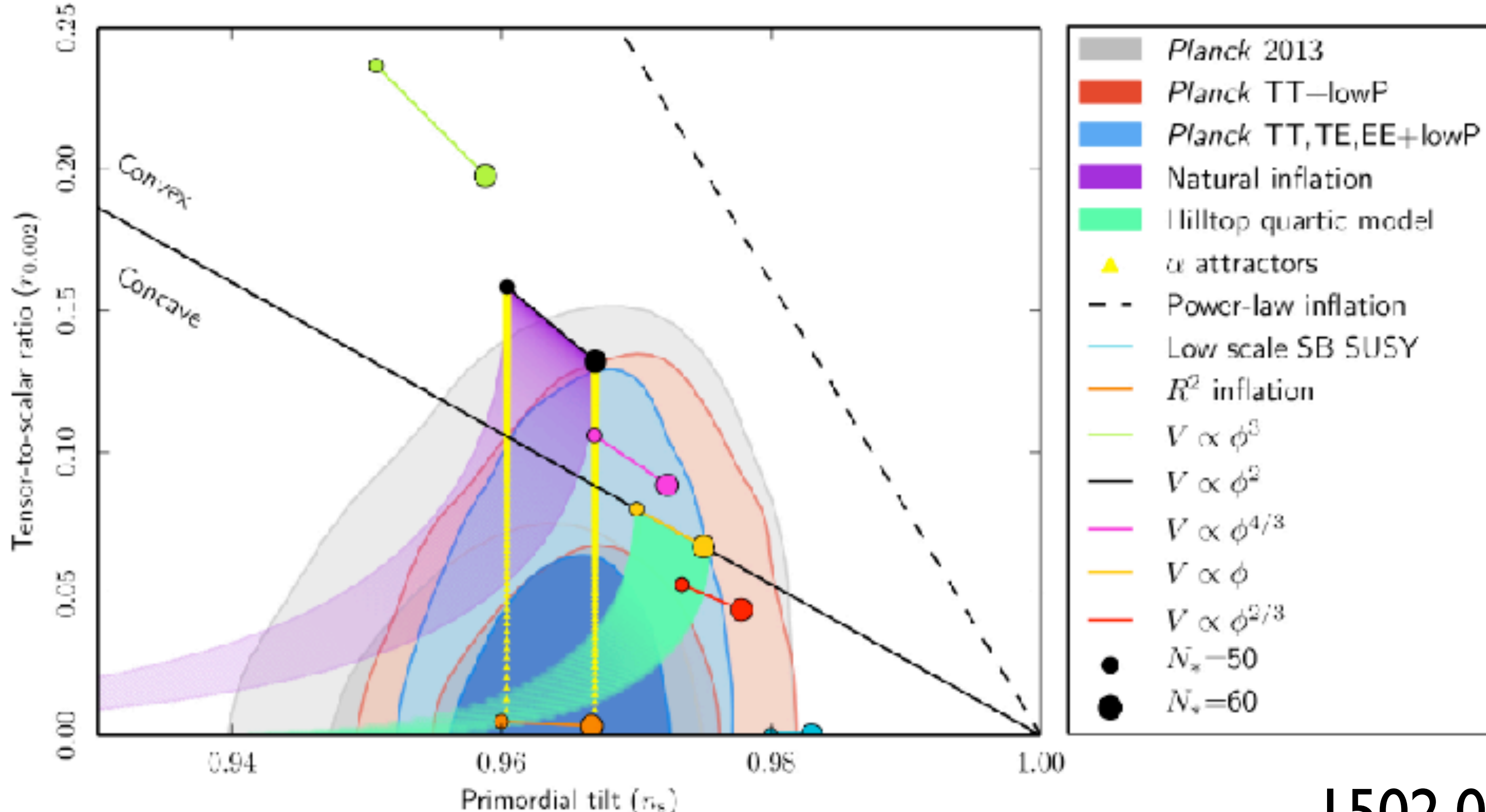
- Slow-roll inflation requires a scalar field with flat potential.

$$n_s = 1 - 6\epsilon + 2\eta \sim 0.965, \quad \epsilon \sim \frac{(V')^2}{V^2} \ll 1, \quad \eta \sim \frac{V''}{V} \ll 1.$$

- Inflaton leads to detectable gravity waves in polarized CMB.

$$r = 16\epsilon \sim 0.1, \quad \epsilon \sim 0.006.$$

Constraints on inflation



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- Almost scale-invariant CMB; no tensor mode.

  n_s < 0.9652±0.0047 (Plank TT+TE,EE+lowP),

  $r_{0.002}$ < 0.10 (Planck TT+lowP, 95% CL)

- Gaussian CMB anisotropies: canonical single field inflation favored. e.g. $f_{NL}^{\rm local} = 0.8 \pm 5.0$ (T & E 68% CL)

Higgs inflation

[Bezrukov, Shaposhnikov (2007)]

- Non-minimal coupling is present in QFT on curved background, modifying the Higgs potential at large fields.

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\mathcal{R} + \boxed{\xi|H|^2\mathcal{R}} - |D_\mu H|^2 - \lambda_H(|H|^2 - v^2/2)^2\Big)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{1}{2}\mathcal{R}(g_E) - \boxed{\frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2} - \frac{1}{\Omega}|D_\mu H|^2 - \boxed{\frac{V}{\Omega^2}}\Big)$$

$$g_{\mu\nu} = \Omega^{-1} g_{E,\mu\nu}, \;\; \Omega = 1 + 2\xi|H|^2$$

Higgs Lagrangian modifies in two important ways.

$2\xi|H|^2 \gg 1:$

Kinetic term, $\mathcal{L}_K \sim -\dfrac{3(\partial_\mu|H|)^2}{|H|^2},$

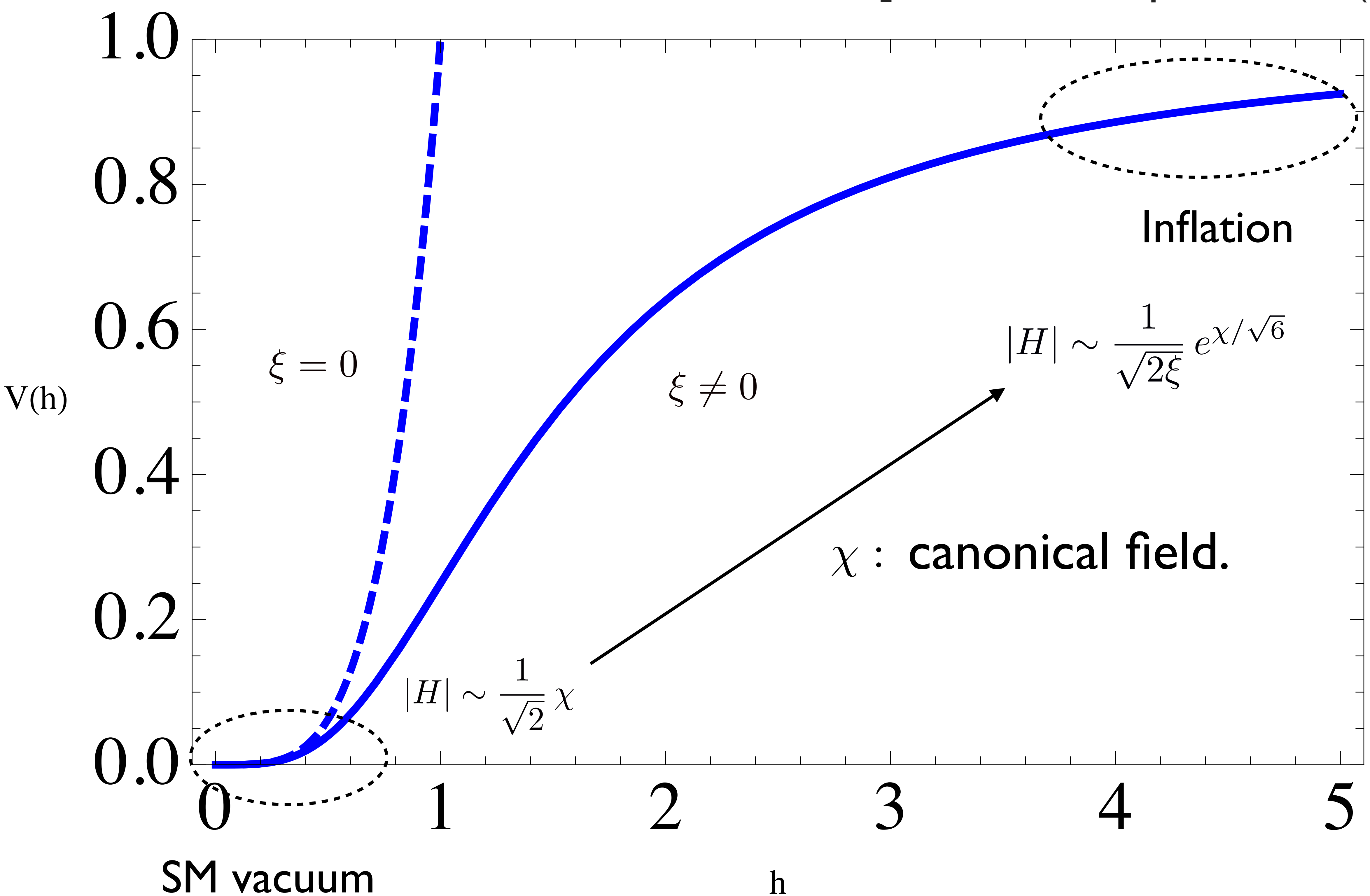
Potential, $V_E \sim \dfrac{\lambda_H}{4\xi^2}\Big(1 - \dfrac{1}{2\xi|H|^2}\Big).$

$\Rightarrow$ Einstein frame potential:

$$V_E \sim \frac{\lambda_H}{4\xi^2}\Big(1 - 2e^{-\chi/\sqrt{6}}\Big).$$

Higgs inflation

[Bezrukov, Shaposhnikov (2007)]



Starobinsky model

- Non-minimal coupling for scalar field is dual to higher curvature term in Starobinsky model.

[Starobinsky (1984)]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{\mathcal{R}}{2} + \frac{1}{2}\xi\phi^2\mathcal{R} - \frac{1}{4}\phi^4\Big) \Longleftrightarrow \mathcal{L} = \sqrt{-g}\Big(\frac{\mathcal{R}}{2} + \frac{1}{4}\xi^2\mathcal{R}\Big)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{R}{2} - \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{4\xi^2}\Big(1 - e^{-2\chi/\sqrt{6}}\Big)^2\Big)$$

in Einstein frame

- Starobinsky-like models are a best fit to Planck data.

$$n_s = 1 - \frac{2}{N} \simeq 0.967, \quad r = \frac{12}{N^2} \simeq 0.0033.$$

- Amplitude of CMB anisotropies: $A_s = \frac{1}{24\pi^2}\frac{V_*}{\epsilon_*} \simeq 2.196 \times 10^{-9}$,

$$\Rightarrow \quad \xi \sim 10^4 \times \sqrt{\lambda_H}$$

Perturbative expansion?

Tree-level unitarity

- Effective interactions in Jordan frame.

[Han, Willenbrock (2004); Burgess, HML, Trott (2009); Barbon, Espinosa (2009)]

Expand fields around classical solution in vacuum,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\omega_2(x) - i\omega_1(x) \\ \underbrace{\varphi(x) + h(x)}_{\phi(x)} + i\omega_3(x) \end{pmatrix}, \qquad g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + \frac{h_{\mu\nu}(x)}{M_P}.$$

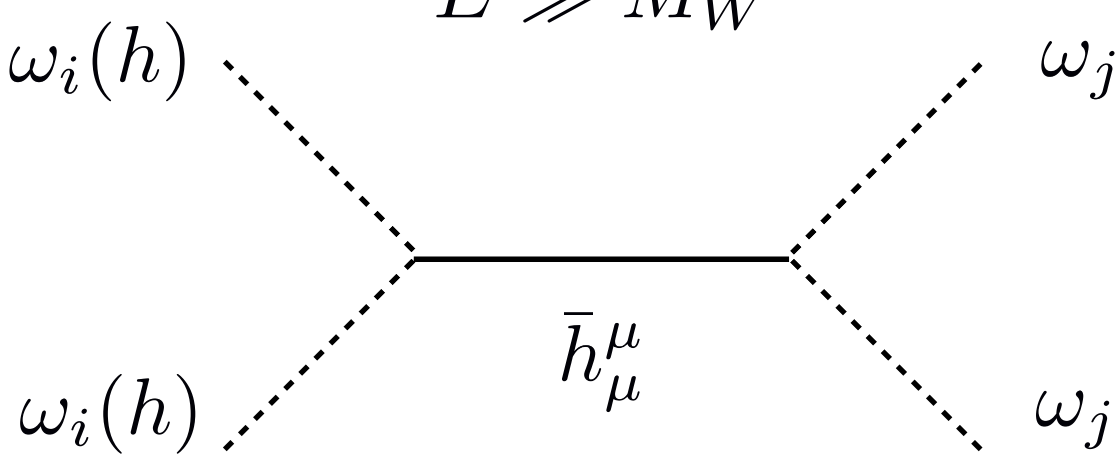
Effective dimension-5 interactions for graviton:

$$\mathcal{L}_{\rm int} = \frac{1}{4}\Big(\omega_1^2 + \omega_2^2 + \omega_3^2 + (\varphi + h)^2\Big)\frac{\xi}{M_P}\Big(\Box \bar{h}^\mu_\mu + 2\partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} + \cdots\Big).$$

- High energy limit:

$E \gg M_W$

Massless Goldstone bosons (Goldstone equivalence theorem)



$$\mathcal{M}_{\omega_i\omega_i \to \omega_j\omega_j} \sim \frac{\xi^2 E^2}{M_P^2},$$

Unitarity bound: $E \lesssim \dfrac{M_P}{\xi}.$

proximate to Hubble scale, $H_{\rm inf} \sim \dfrac{\sqrt{\lambda_H} M_P}{\xi}.$

Tree-level unitarity

- Effective interactions in Einstein frame

Leading dim-6 interactions:

[Burgess, HML, Trott (2010); Hertzberg(2010)]

$$\mathcal{L}_{\rm eff} = \frac{2\xi}{M_P^2}\,|H|^2|D_\mu H|^2 - \boxed{\frac{3\xi^2}{M_P^2}}\,\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) + \frac{4\lambda_H\xi}{M_P^2}\,|H|^6 + \cdots$$

$$-\frac{3\xi^2}{4M_P^2}\,\phi^2(\partial_\mu\phi)^2 - \frac{3\xi^2}{2M_P^2}\,(\omega_3\partial_\mu\omega_3 + \omega_+\partial_\mu\omega_- + {\rm h.c.})\phi\partial^\mu\phi$$

$$-\frac{3\xi^2}{4M_P^2}\,(\omega_3\partial_\mu\omega_3 + \omega_+\partial_\mu\omega_- + {\rm h.c.})^2.$$

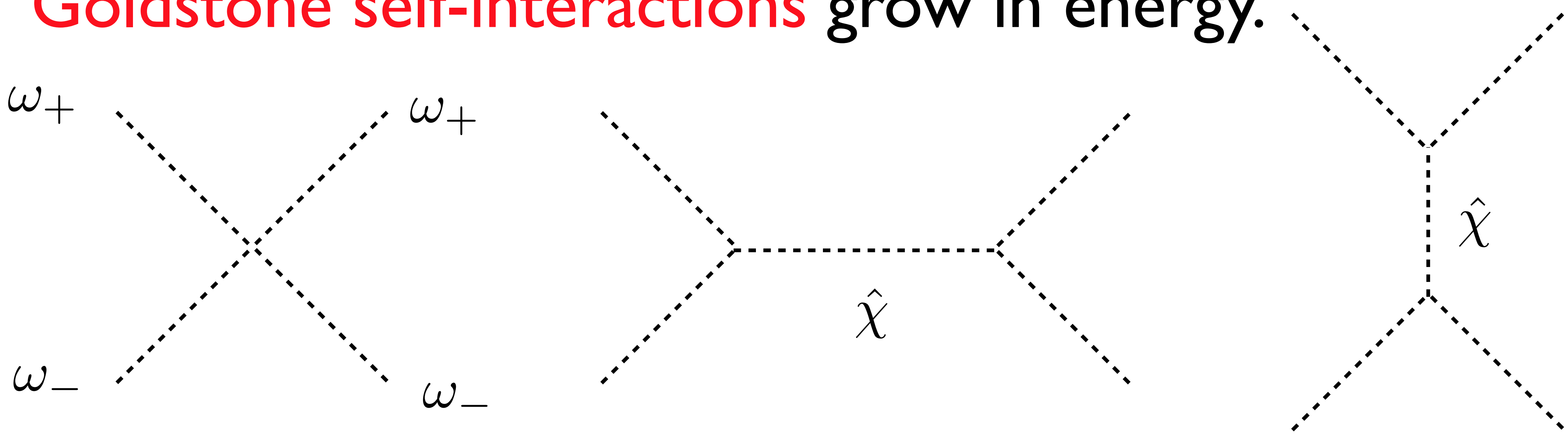
$$\omega_\pm \equiv \frac{1}{\sqrt{2}}(\omega_1 \pm i\omega_2)$$

absorbed by Higgs redefinition:

$$\frac{d\chi}{d\phi} \simeq \sqrt{1 + \frac{3\xi^2\phi^2}{2M_P^2}} \quad \Longrightarrow \quad \boxed{\phi = \chi - \frac{\xi^2}{4M_P^2}\,\chi^3 \cdots}$$

Tree-level unitarity

- Goldstone self-interactions grow in energy.



$$\mathcal{M}_{\omega_+\omega_-\to\omega_+\omega_-} \simeq -\frac{3i\xi^2}{2M_P^2}\, t \quad \Rightarrow \quad a_0 = \frac{1}{16\pi s}\int_{-s}^{0} |\mathcal{M}|dt \simeq \frac{3\xi^2 s}{32\pi M_P^2} < \frac{1}{2},$$

Unitarity bound: $E \lesssim \sqrt{\frac{16\pi}{3}}\,\frac{M_P}{\xi}.$

cf. Unitary gauge: Burgess, HML, Trott (2010).

- Dim-6 Higgs self-interactions also violate unitarity.

$$\boxed{\phi = \chi - \frac{\xi^2}{4M_P^2}\chi^3 \cdots} \quad \Rightarrow \quad V_E \simeq \frac{1}{4}\lambda_H \chi^4 - \boxed{\frac{\lambda_H \xi^2}{M_P^2}}\chi^6 + \cdots$$

Field-dependent cutoff

- During inflation, the cutoff-scale is field-dependent.

$$2\xi|H|^2 \gg 1: \qquad -\frac{3\xi^2}{\Omega^2}(\partial_\mu |H|^2)^2 \to -\frac{3}{4}\boxed{\frac{1}{|H|^2}}(\partial_\mu |H|^2)^2$$

$$V_E = \frac{V}{\Omega^2} \to \frac{\lambda_H}{4\xi^2}\left(1 - \boxed{\frac{1}{\xi|H|^2}} + \cdots\right)$$

$\Rightarrow$ Suppression scale of higher dimensional operators in this expansion is field-dependent.

- Higgs interactions are exponentially suppressed:

$$(\xi|H|^2)^{-n} \sim e^{-2n\chi/\sqrt{6}}, \qquad \chi: \text{ canonical field for Higgs modulus.}$$

[Linde et al (2010)]

- Massive gauge theory sets the larger cutoff scale.

$$M_W^2 = \frac{g^2|H|^2}{2\Omega} \to \frac{g^2 M_P^2}{4\xi}$$

$\Rightarrow$ cutoff: $\Lambda_{\rm inf} \sim \frac{M_P}{\sqrt{\xi}} > \frac{M_P}{\xi}.$

[Bezrukov, Shaposhnikov et al (2010)]

Validity of Higgs inflation

- Field-dependent is much larger than Hubble scale.

$$\mathcal{L}_{\mathcal{R}} = \mathcal{R} + c_2\mathcal{R}^2 + c_3\frac{\mathcal{R}^3}{\Lambda^2} + \cdots$$
$$\sim H_{\rm inf}^2\left(1 + c_2 H_{\rm inf}^2 + c_3\frac{H_{\rm inf}^3}{\Lambda_{\rm inf}^2} + \cdots\right)$$

$H_{\rm inf} < \Lambda_{\rm inf}$: Curvature expansion is ok.

- Field expansion during inflation is still in question.

$\phi_{\rm inf} \gg \Lambda_{\rm inf}$: $$\mathcal{L}_{\phi} = \left(\frac{\phi}{\Lambda_{\rm inf}}\right)^n (\partial_\mu\phi)^2 + \frac{\phi^{m+4}}{\Lambda_{\rm inf}^m}$$?

~ Trans-Planckian fields in chaotic inflation.

- High momentum limit near vacuum such as reheating process is still in question.

Quantum effects

[Salopek et al (1989)]

- Jordan frame action for inflaton and graviton.

$$ds^2 = a^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu \; : \; \mathcal{L}_J = \sqrt{-g}\Big(f(\phi)\mathcal{R} - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\Big)$$

$$\to -\frac{1}{2}a^2(\partial_\mu\phi)^2 + 6f(\partial_\mu a)^2 + 6a\frac{df}{d\phi}(\partial_\mu a)(\partial^\mu\phi) - a^4V.$$

- Canonical quantization conditions:

$$[\phi(\vec{x},\tau), P_\phi(\vec{x}',\tau)] = i\delta^3(\vec{x}-\vec{x}'),$$

$$[a(\vec{x},\tau), P_a(\vec{x}',\tau)] = i\delta^3(\vec{x}-\vec{x}'),$$

$$\Rightarrow \quad [\phi(\vec{x},\tau), \dot{\phi}(\vec{x}',\tau)] = is(\phi)\delta^3(\vec{x}-\vec{x}'), \quad \boxed{s(\phi) \equiv \frac{fa^{-2}}{f+3f'^2}.}$$

Inflaton propagator: $\langle 0|\mathrm{T}\phi(x)\phi(y)|0\rangle = \boxed{s(\phi)}\int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\epsilon}\,e^{-ip\cdot(x-y)}\,.$

$$f = \frac{1}{2}(1+\xi\phi^2) \quad \Rightarrow \quad s(\phi) = \frac{1+\xi\phi^2}{1+\xi(1+6\xi)\phi^2} \sim \frac{1}{6\xi}, \quad \phi \gg 1/\sqrt{\xi}.$$

inflaton loops suppressed!

RG equations

[Wilczek et al (2008); McDonald et al (2011);HML (2013)]

- Higgs loops are isolated in RG equations for the SM.

$$(4\pi)^2\frac{d\lambda_H}{d\ln\mu} = (12y_t^2 - 3g'^2 - 9g^2)\lambda_H - 6y_t^4 + \frac{3}{8}\left(2g^4 + (g'^2+g^2)^2\right) + (18c_h^2 + 6)\lambda_H^2$$

$$(4\pi)^2\frac{dg'}{d\ln\mu} = \frac{81+c_h}{12}g'^3 + \frac{g'^3}{16\pi^2}\left(\frac{199g'^2}{18} + \frac{9g^2}{2} + \frac{44g_3^2}{3} - \frac{17c_hy_t^2}{6}\right),$$

$$(4\pi)^2\frac{dg}{d\ln\mu} = -\frac{39-c_h}{12}g^3 + \frac{g^3}{16\pi^2}\left(\frac{3}{2}g'^2 + \frac{35}{6}g^2 + 12g_3^2 - \frac{3}{2}c_hy_t^2\right),$$

$$(4\pi)^2\frac{dg_3}{d\ln\mu} = -7g_3^3 + \frac{g^3}{16\pi^2}\left(\frac{11}{6}g'^2 + \frac{9}{2}g^2 - 26g_3^2 - 2c_hy_t^2\right),$$

$$c_h \equiv \frac{1+\xi h^2}{1+\xi(1+6\xi)h^2}$$

$$(4\pi)^2\frac{dy_t}{d\ln\mu} = y_t\left(\left(\frac{23}{6}+\frac{2}{3}c_h\right)y_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$
$$+\frac{y_t}{16\pi^2}\Big[-\frac{23}{4}g^4 - \frac{3}{4}g^2g'^2 + \frac{1187}{216}g'^4 + 9g^2g_3^2$$
$$+\frac{19}{9}g'^2g_3^2 - 108g_3^4 + c_hy_t^2\left(\frac{225}{16}g^2 + \frac{131}{16}g'^2 + 36g_3^2\right)$$
$$+6(-2c_h^2y_t^4 - 2c_h^3y_t^2\lambda_H + c_h^2\lambda_H^2)\Big].$$

$$(4\pi)^2\frac{d\xi}{d\ln\mu} = \left((6+6c_h)\lambda_H + 6y_t^2 - \frac{3}{2}(3g^2+g'^2)\right)\left(\xi+\frac{1}{6}\right)$$

Running QCD and top Yukawa couplings modified during inflation!

UV complete Higgs inflation

[Giudice,HML (2010)]

- Higgs kinetic term in Einstein frame is a type of non-linear sigma model, like pion kinetic terms.

$$\mathcal{L}_{\rm kin} = -\frac{1}{2(1+\xi_0\vec{\phi}^2)}\left(\delta_{ij}+\frac{6\xi_0^2\phi_i\phi_j}{1+\xi_0\vec{\phi}^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j.$$

- It is linearized by introducing a sigma field with "non-compact" field space, $\sigma^2-\vec{\phi}^2=\xi_0^{-1}$,

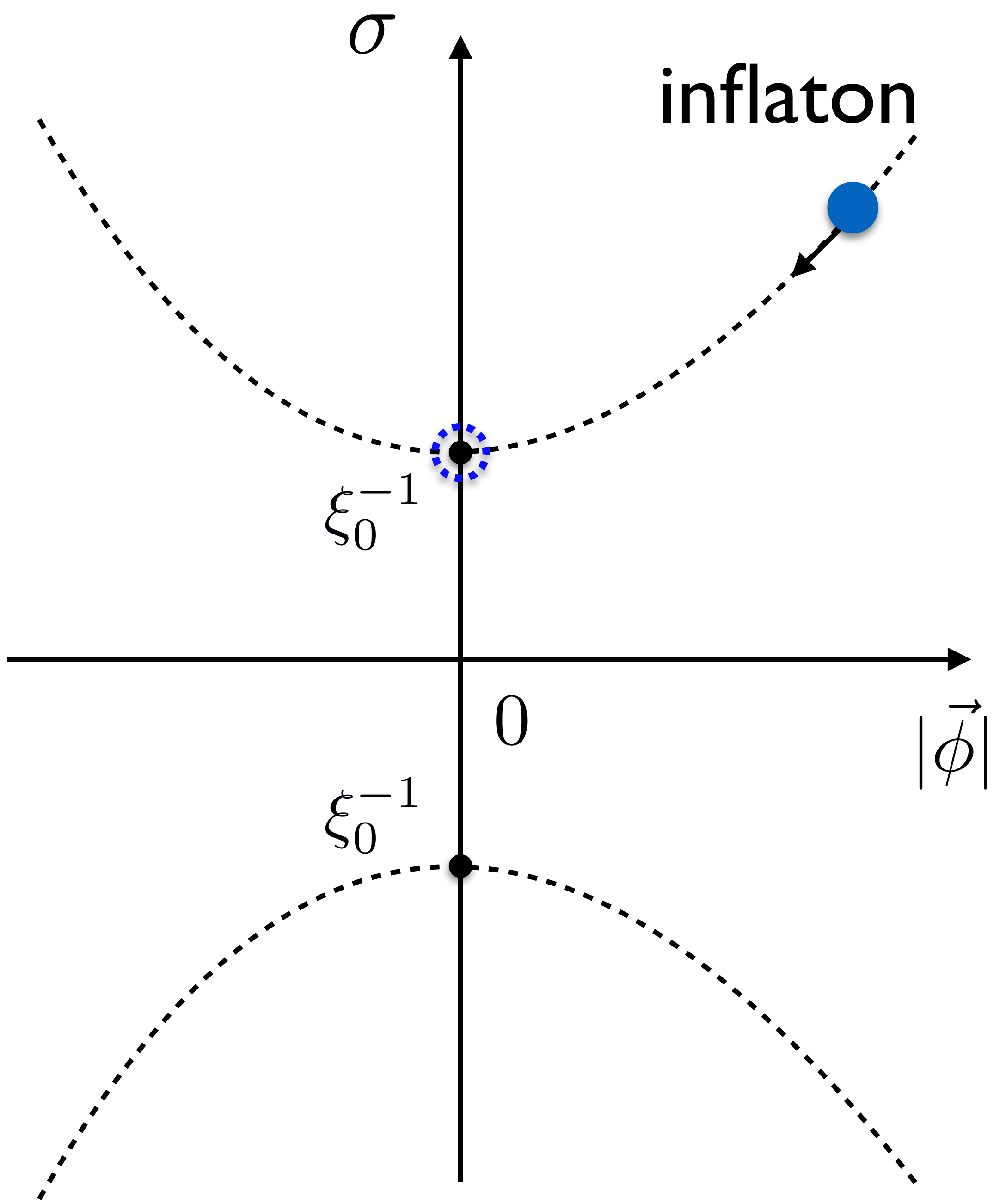
$$\mathcal{L}_{\rm kin} = -\frac{1}{2\xi_0\sigma^2}\left[(\partial_\mu\vec{\phi})^2+6\xi_0(\partial_\mu\sigma)^2\right]:\ SO(1,5)/SO(5).$$

- The model is equivalent to "induced gravity" model with Higgs couplings in Jordan frame.

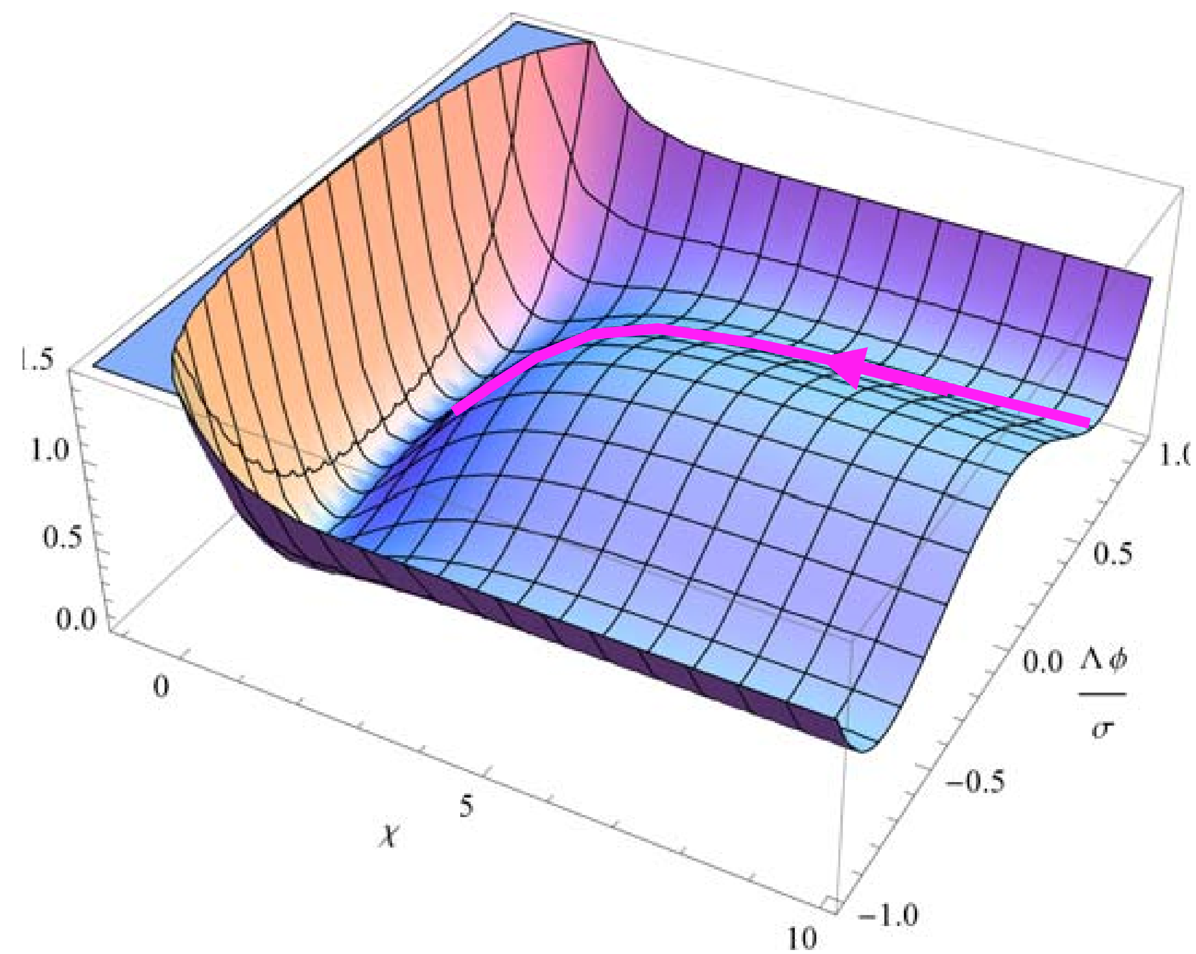
$$\mathcal{L}_J=\sqrt{-g}\left[\underbrace{\xi_0\sigma^2\mathcal{R}}_{\text{"induced" Einstein term}}-\frac{1}{2}(\partial_\mu\sigma)^2-\lambda\left(\sigma^2-\vec{\phi}^2-\xi_0^{-1}\right)^2\right]$$

Dynamics of sigma model

[Giudice, HML (2010)]



$SO(1,5)/SO(5)$



Higgs-sigma mixed inflation

Predictions of Higgs inflation are maintained.

General UV completion

[Giudice, HML (2014)]

- Consider a general induced gravity model.

$$\mathcal{L} = \sqrt{-g}\left[\frac{\Omega(\phi)}{2}R - \frac{K(\phi)}{2}(\partial\phi)^2 - U(\phi)\right]$$

$$K = 1, \quad \Omega = \xi f(\phi), \quad U = \lambda[f(\phi) - \xi^{-1}]^2$$

$\Rightarrow$ $M_P^2 \sim \xi f(\langle\phi\rangle)$ due to large inflaton VEV.

- Large inflaton VEV maintains a Planck-scale cutoff.

- Introduce a Higgs coupling, $\mathcal{L}_{\rm int} = \lambda_m[f(\phi) - \xi^{-1}]|H|^2.$

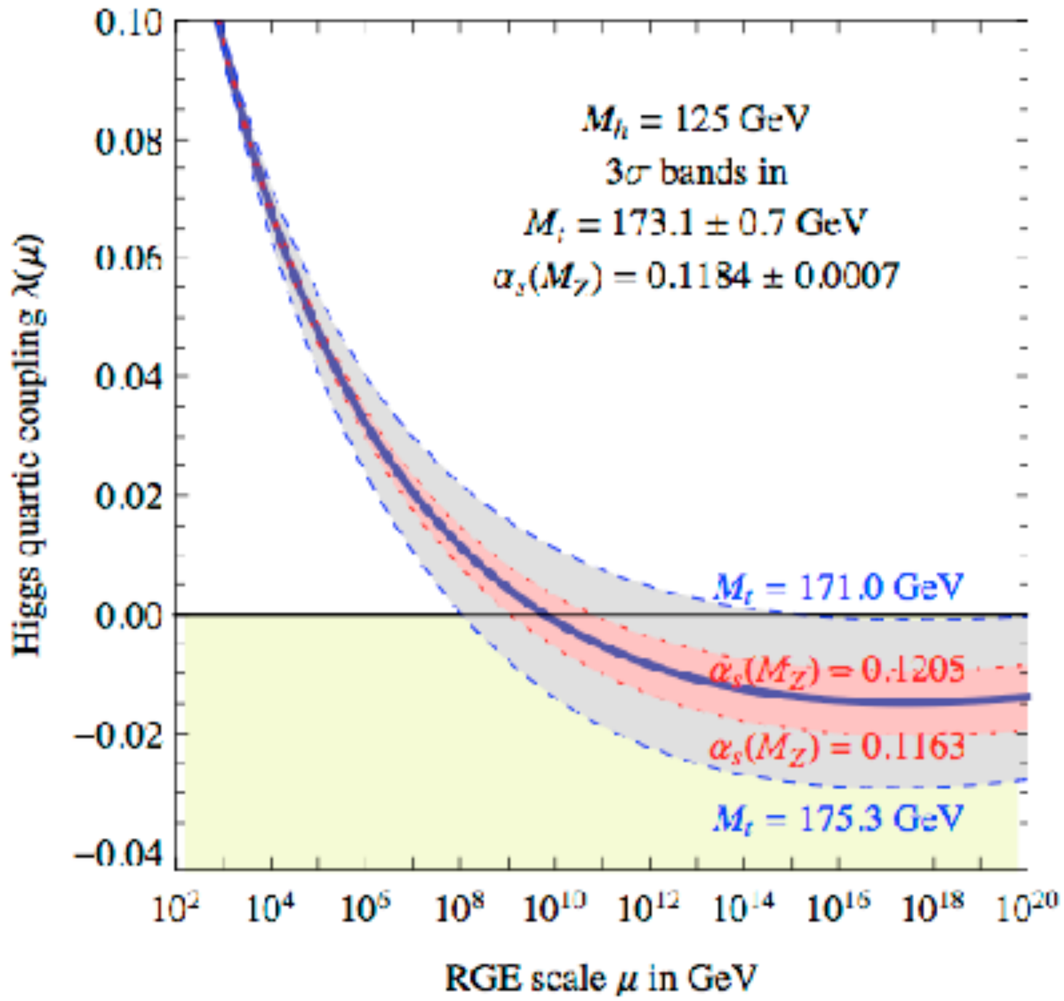
  Effective non-minimal and quartic couplings for Higgs:

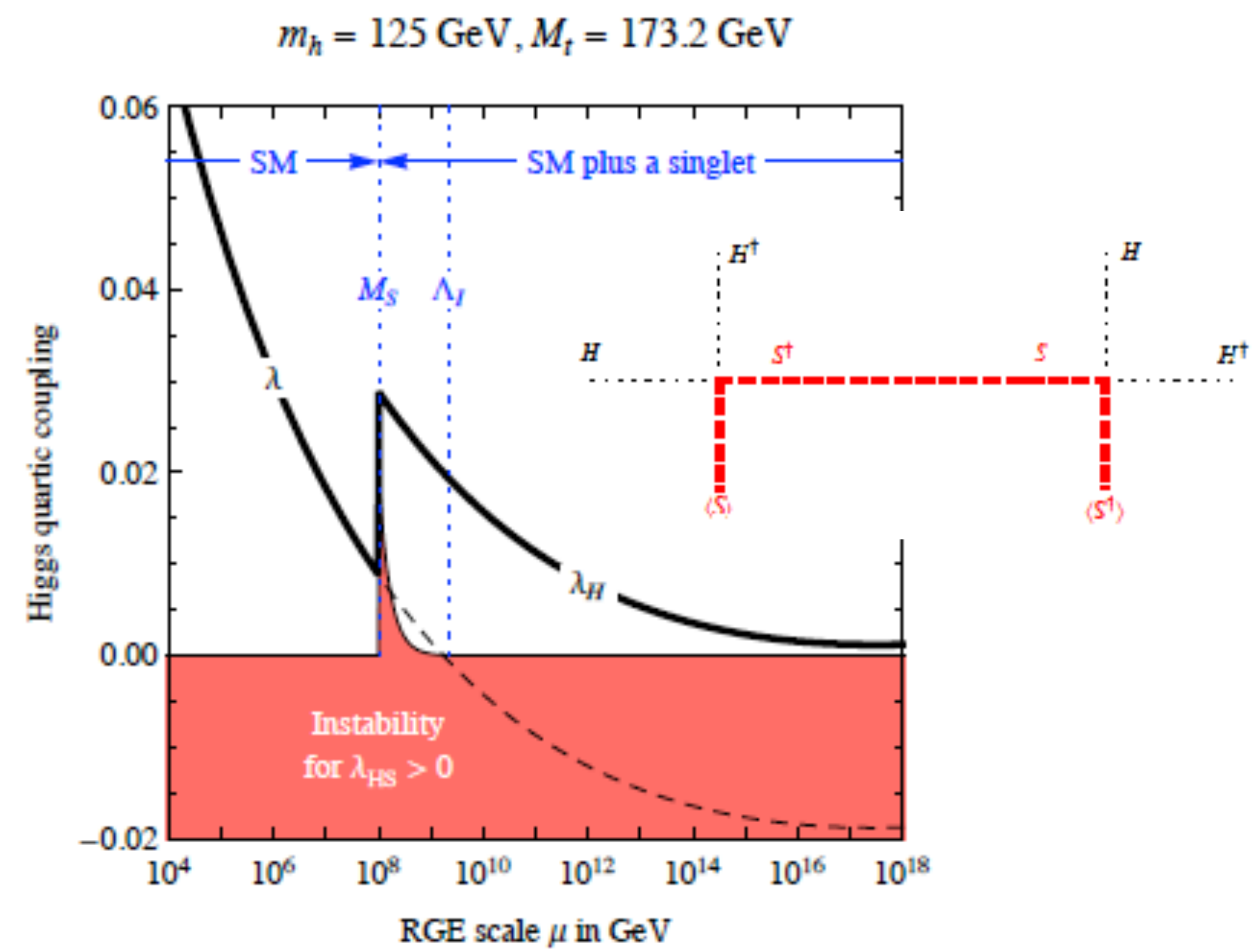
$\Rightarrow$ $$\mathcal{L}_{\rm eff} = \frac{1}{2}\Big(1 + 2\xi_{\rm eff}|H|^2\Big)\mathcal{R} - \frac{1}{4}\lambda_{\rm eff}|H|^4,$$

$$\xi_{\rm eff} = \frac{\xi\lambda_m}{4\lambda}, \quad \lambda_{\rm eff} = \lambda_H - \frac{\lambda_m^2}{\lambda}.$$

Vacuum stability

[Giudice, HML, Espinosa et al (2012)]





- Higgs mass needs a small quartic coupling: $m_h^2 = 2v^2\lambda_H$.

$\lambda_H(m_H) = 0.13$ ⟶ SM vacuum might be unstable.

- Mixing with heavy scalar leads to a shifted quartic coupling.

$m_h^2 = 2v^2\left(\lambda_H - \frac{\lambda_m^2}{\lambda}\right)$ ⟶ $\lambda_H = 0.13 + \frac{\lambda_m^2}{\lambda}$ extra quartic couplings

Conclusions

- Non-minimal coupling is necessary in renormalized theory on curved space, so it should be taken into account for inflation dynamics.
- Large non-minimal coupling leads to premature violation of unitarity via momentum-dependent Goldstone boson scatterings (dim-5 operator for Higgs-graviton or dim-6 Higgs self-interactions.).
- The UV cutoff during inflation becomes larger than the one in vacuum, so semi-classical approximation may be justified.
- Induced gravity might have played a role in early Universe to cure the Higgs inflation and vacuum.