Magnetic Field Calculation in Cross Calibration

Cryogenics Science Center, KEK
Hiroshi Yamaguchi

Items

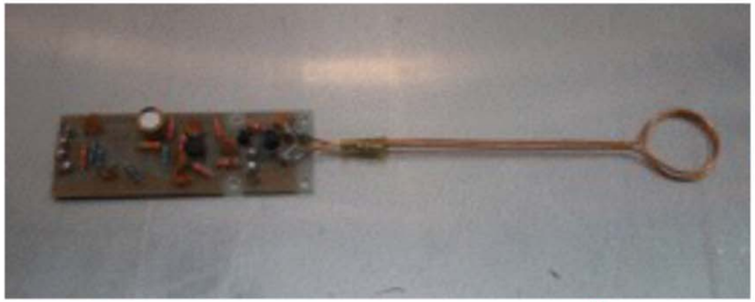
- Calculation magnetic field in the latest cross calibration test
- Comparison material effect between measurement and calculation
- Preparations for next cross calibration test

Cross Calibration

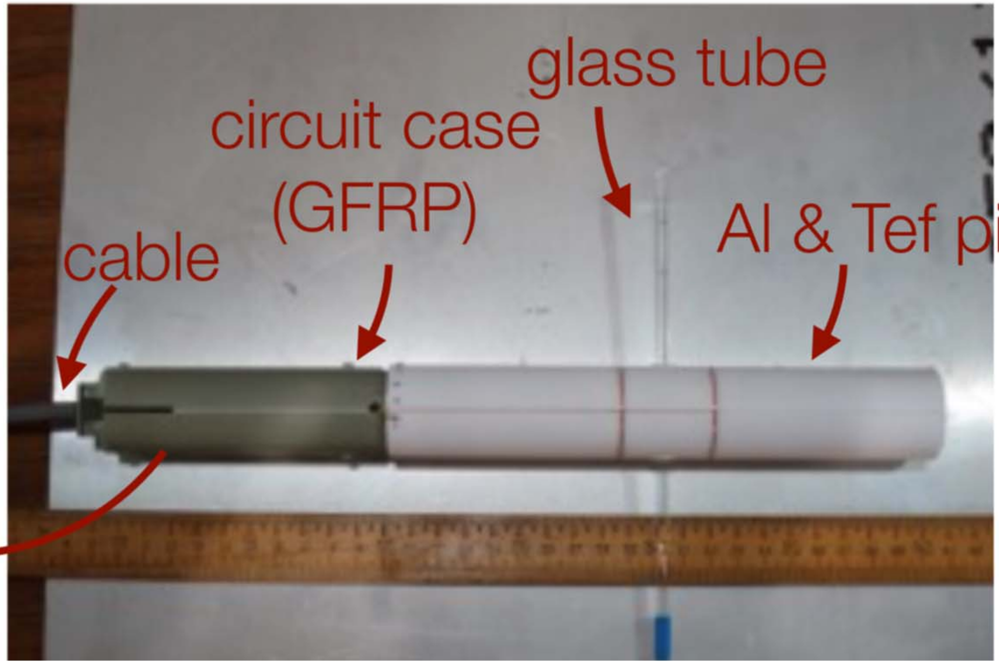
The latest cross calibration test was done in March 2017

- Thin and long glass tube (fill water in)
- Using Al pipe and Teflon pipe to support the glass tube and modulation coils as well as to cancel the error magnetic field
- Improve electrical circuit

circuit board & RF coil



inside



Motivations

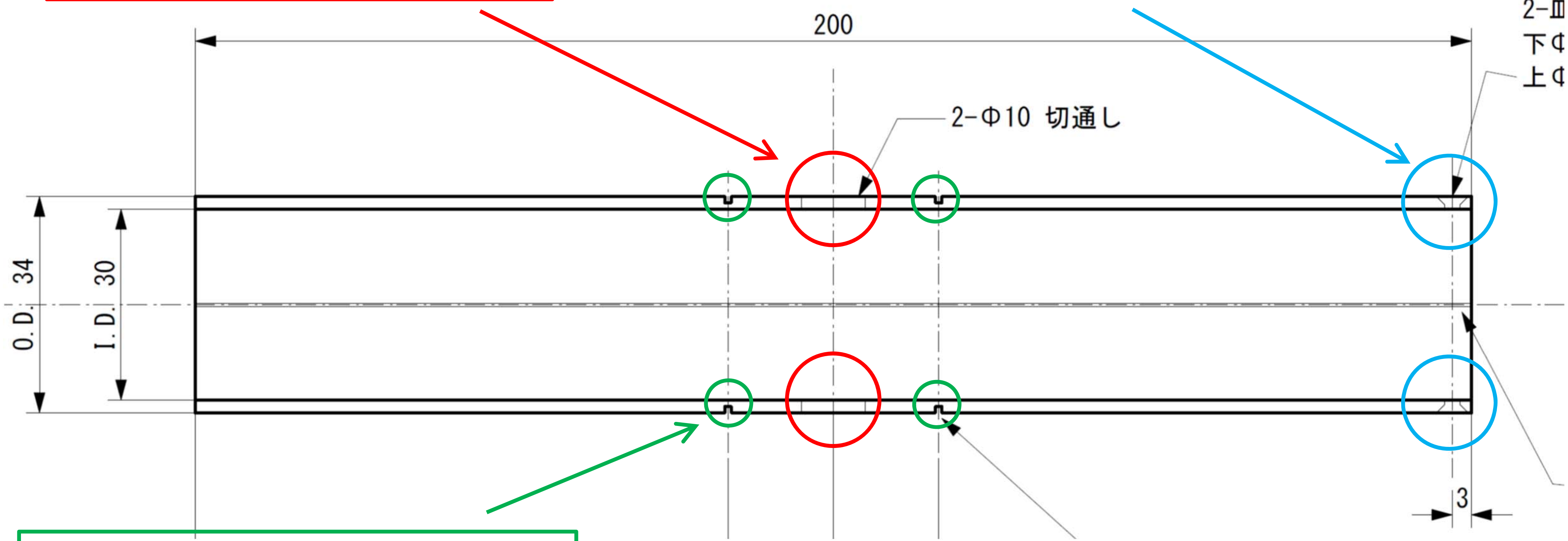
- Reconstruct the cross calibration results
- Reduce the material effects for next cross calibration test

Geometry (Al pipe and Teflon pipe)

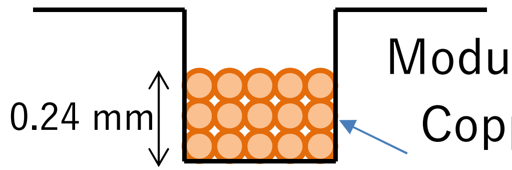
Standard probe is consist of an Al pipe (1 mm thickness) and a Teflon pipe (2 mm thickness)

Center holes (to fix a glass tube)
Al pipe : ϕ8.2
Teflon pipe : ϕ10

holes
Al pipe : ϕ2.2
Teflon pipe : ϕ2.4



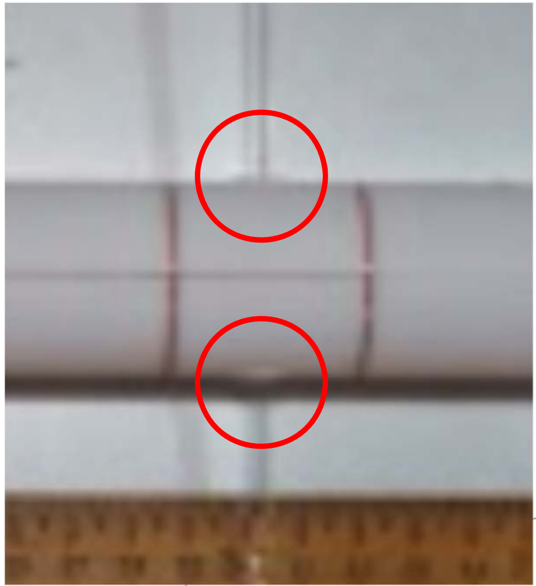
Slits (only Teflon pipe)
Width 1 mm, depth 1 mm
Winding modulation coils in this slits

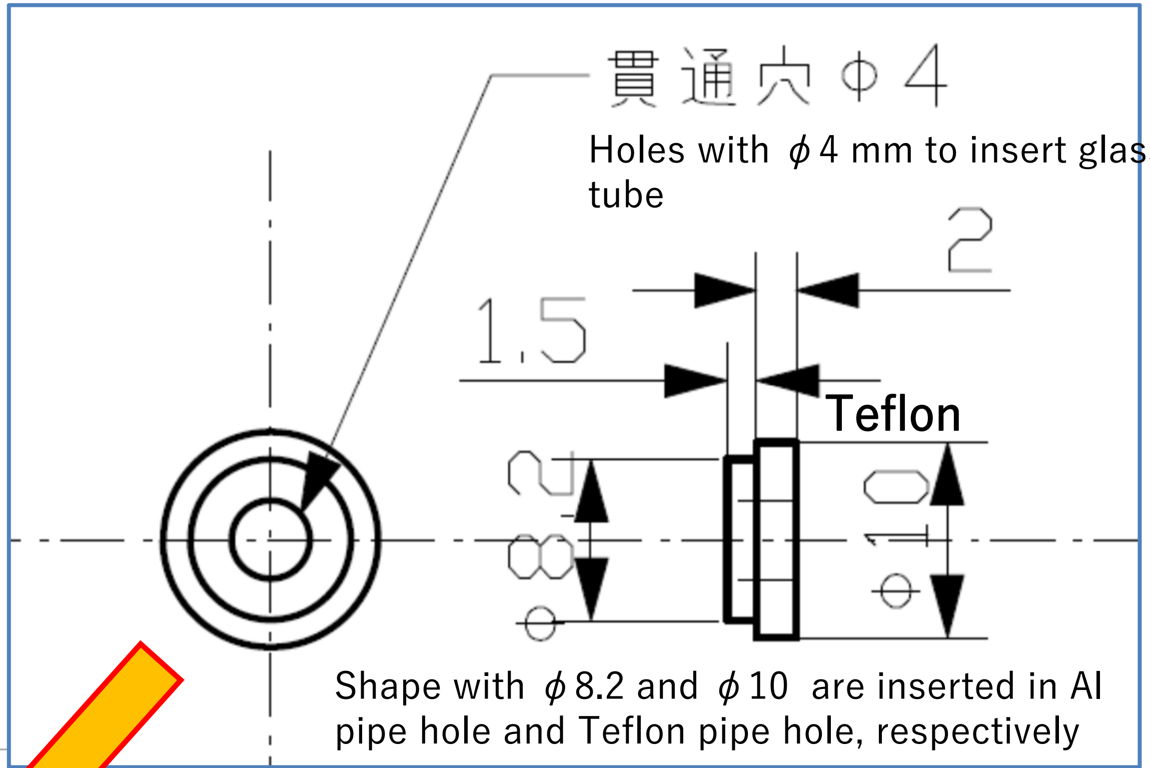


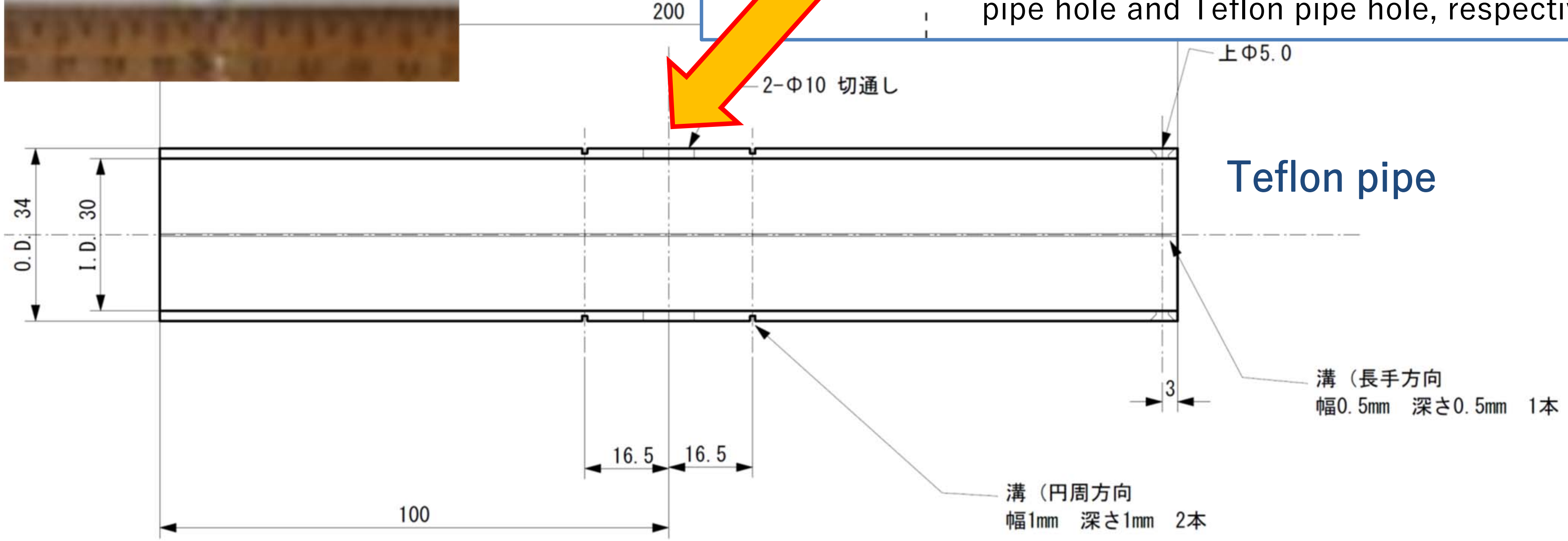
Modulation coil
Copper wire : ϕ0.1 mm, 30 turns
Cross section of coils : 0.24 mm^2
It corresponds to 0.24 mm depth

Stopper for Glass Tube

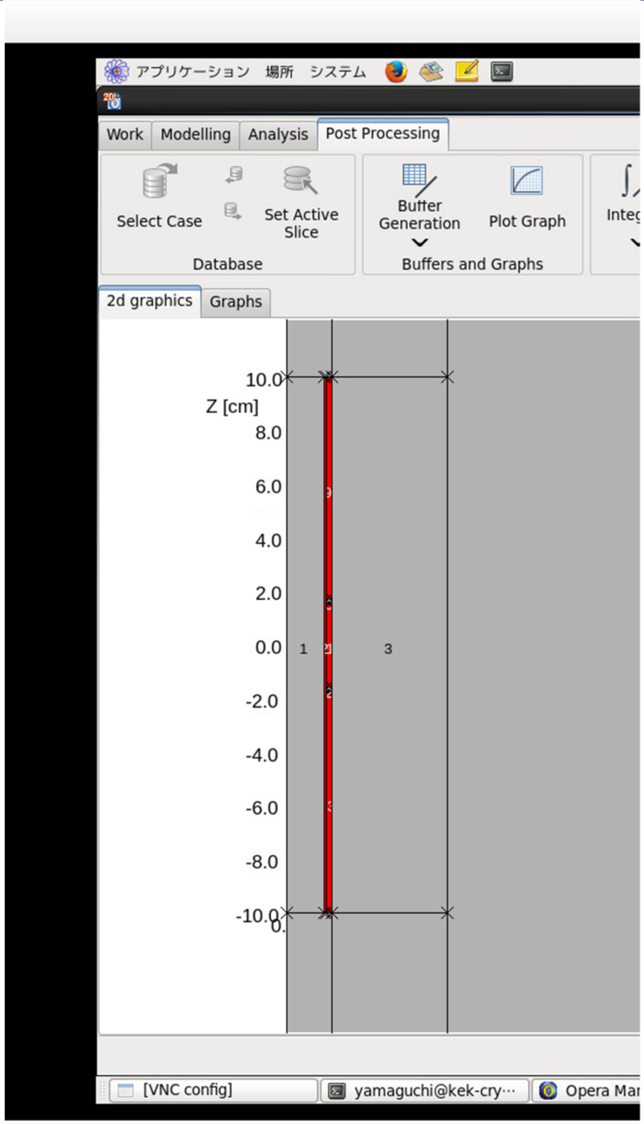
Glass tube filled with water is fixed by using stopper made of Teflon

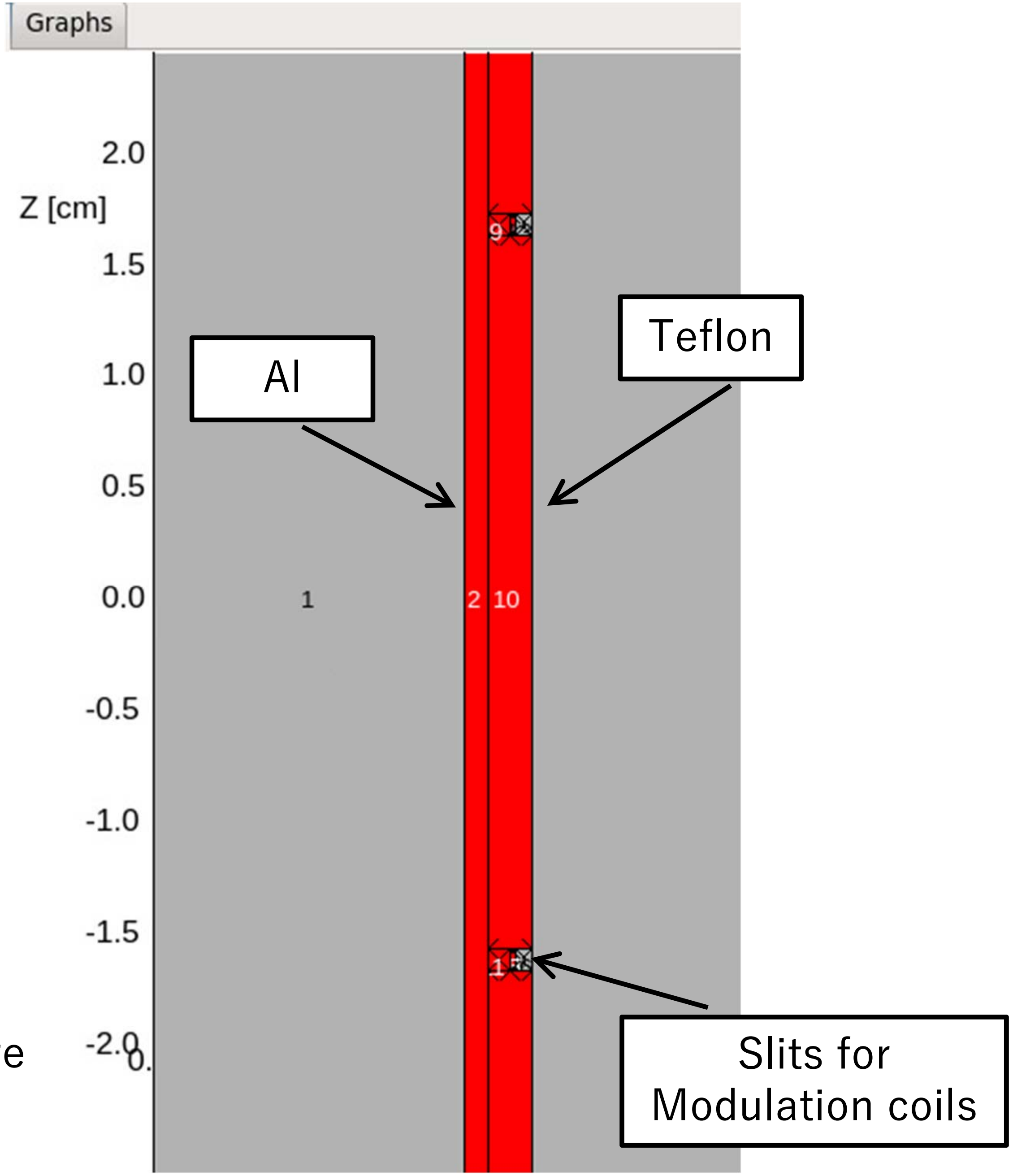






Geometry (Opera)





Mesh size

Standard probe and near atmosphere region : 0.5 mm

Outer atmosphere region : 10 mm

My program

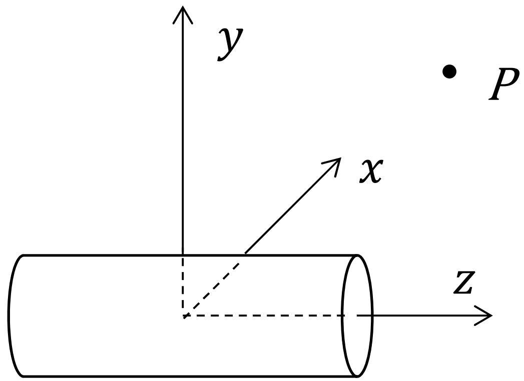
Write a program for magnetic field calculation by using C++ language

Pro : It is easy for changing the geometry

Con : The calculation can not be included a self-consist effect

- At first, the magnetic fields are calculated by using my program to improve the error field
- Finally, it is confirmed by using Opera

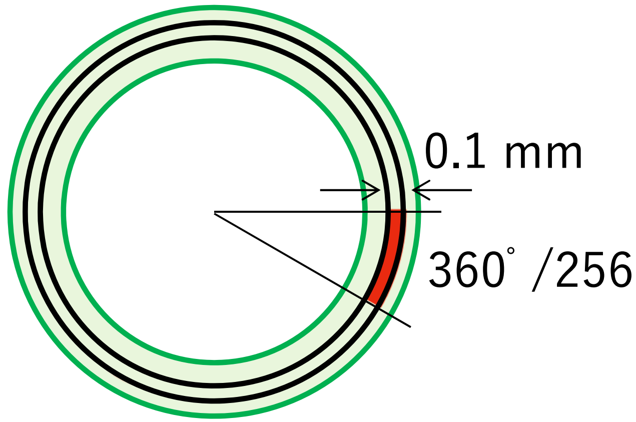
1. Generate meshes in pipes
2. Calculate magnetic flux between an observed point (point P) and meshes
3. Sum contributions from all meshes



Mesh profile

Cylindrical Mesh : dr = 0.1 mm, d $\theta = 2\pi/256$, dz = 1 mm

Magnetic field : 1.0 Tesla



Field Calculation

$\vec{H}_0$: Magnetic Field generated by MRI Coil

$$\vec{H}_0[\mathrm{A}\cdot\mathrm{m}^{-1}] = (0, 0, H_0) = \left(0, 0, \frac{B_0[\mathrm{Wb}\cdot\mathrm{m}^{-2}]}{\mu_0[\mathrm{Wb}\cdot\mathrm{A}^{-1}\cdot\mathrm{m}^{-1}]}\right)$$

$\vec{H}_M$: Magnetic Field generated by material with magnetic susceptibility χ_M in H_0

Magnetization : $\vec{M}[\mathrm{Wb}\cdot\mathrm{m}^{-2}]$ ($\vec{m}[\mathrm{Wb}\cdot\mathrm{m}]$: magnetic moment in unit volume V)

$$\vec{M}[\mathrm{Wb}\cdot\mathrm{m}^{-2}] = \frac{\vec{m}[\mathrm{Wb}\cdot\mathrm{m}]}{V[\mathrm{m}^3]} = \mu_0[\mathrm{Wb}\cdot\mathrm{A}^{-1}\cdot\mathrm{m}^{-1}]\chi_\mathrm{M}\vec{H}[\mathrm{A}\cdot\mathrm{m}^{-1}]$$

Magnetic field generated by Magnetizations

$$H_r[\mathrm{A}\cdot\mathrm{m}^{-1}\cdot\mathrm{m}^{-3}] = \frac{M[\mathrm{Wb}\cdot\mathrm{m}^{-2}]}{2\pi\mu_0[\mathrm{Wb}\cdot\mathrm{A}^{-1}\cdot\mathrm{m}^{-1}]r^3[\mathrm{m}^3]}\cos\theta$$

$$H_\theta[\mathrm{A}\cdot\mathrm{m}^{-1}\cdot\mathrm{m}^{-3}] = \frac{M[\mathrm{Wb}\cdot\mathrm{m}^{-2}]}{4\pi\mu_0[\mathrm{Wb}\cdot\mathrm{A}^{-1}\cdot\mathrm{m}^{-1}]r^3[\mathrm{m}^3]}\sin\theta$$

Covert Spherical coordinates to rectangular coordinate

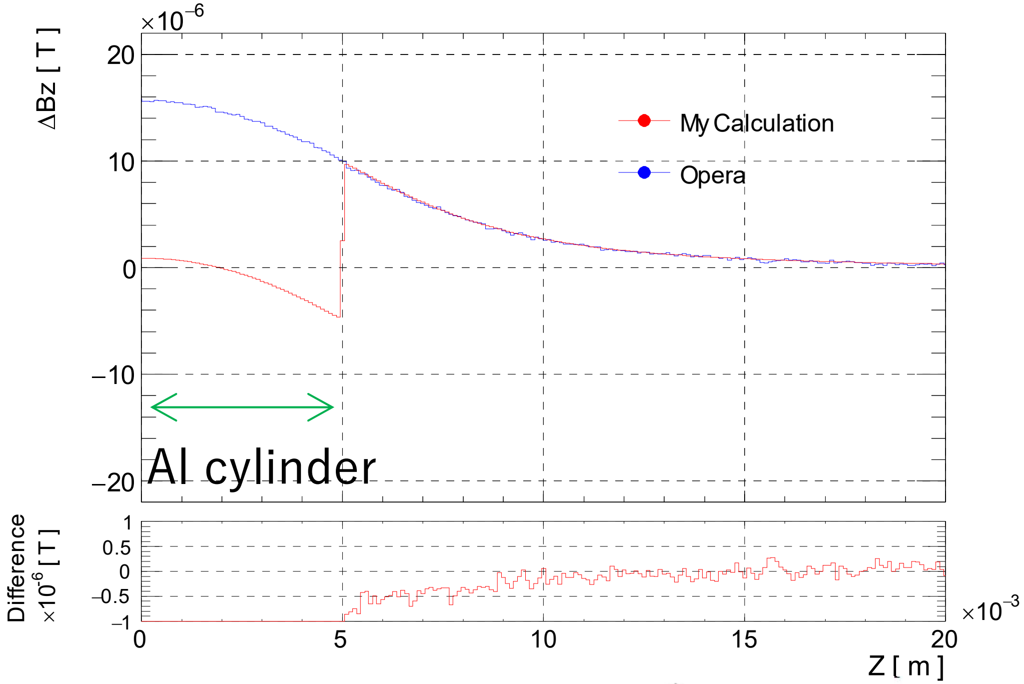
$$\vec{H}_M = \begin{pmatrix} H_r\sin\theta\cos\phi + H_\theta\cos\theta\cos\phi \\ H_r\sin\theta\sin\phi + H_\theta\cos\theta\sin\phi \\ H_r\cos\theta - H_\theta\sin\theta \end{pmatrix}$$

Magnetic field (magnetic flux density) generated by material

$$\vec{B}_M[\mathrm{Wb}\cdot\mathrm{m}^{-2}] = \mu[\mathrm{Wb}\cdot\mathrm{A}^{-1}\cdot\mathrm{m}^{-1}]\int \vec{H}_M[\mathrm{A}\cdot\mathrm{m}^{-1}\cdot\mathrm{m}^{-3}]dV$$

Compare the magnetic field between my program and Opera by using Al cylinder (Height 10 mm, Diameter 10 mm)

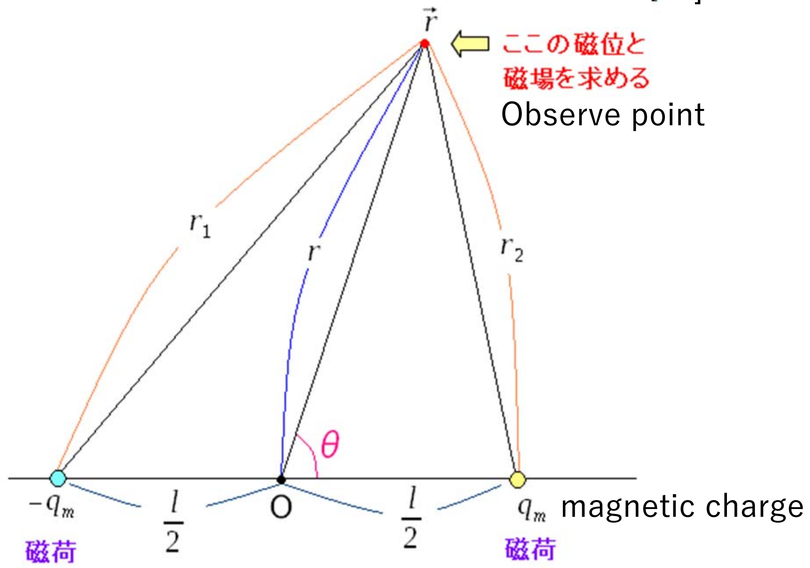
My program is not consistent with Opera in materials



Approximation (r >> ℓ) is applied in my calculation

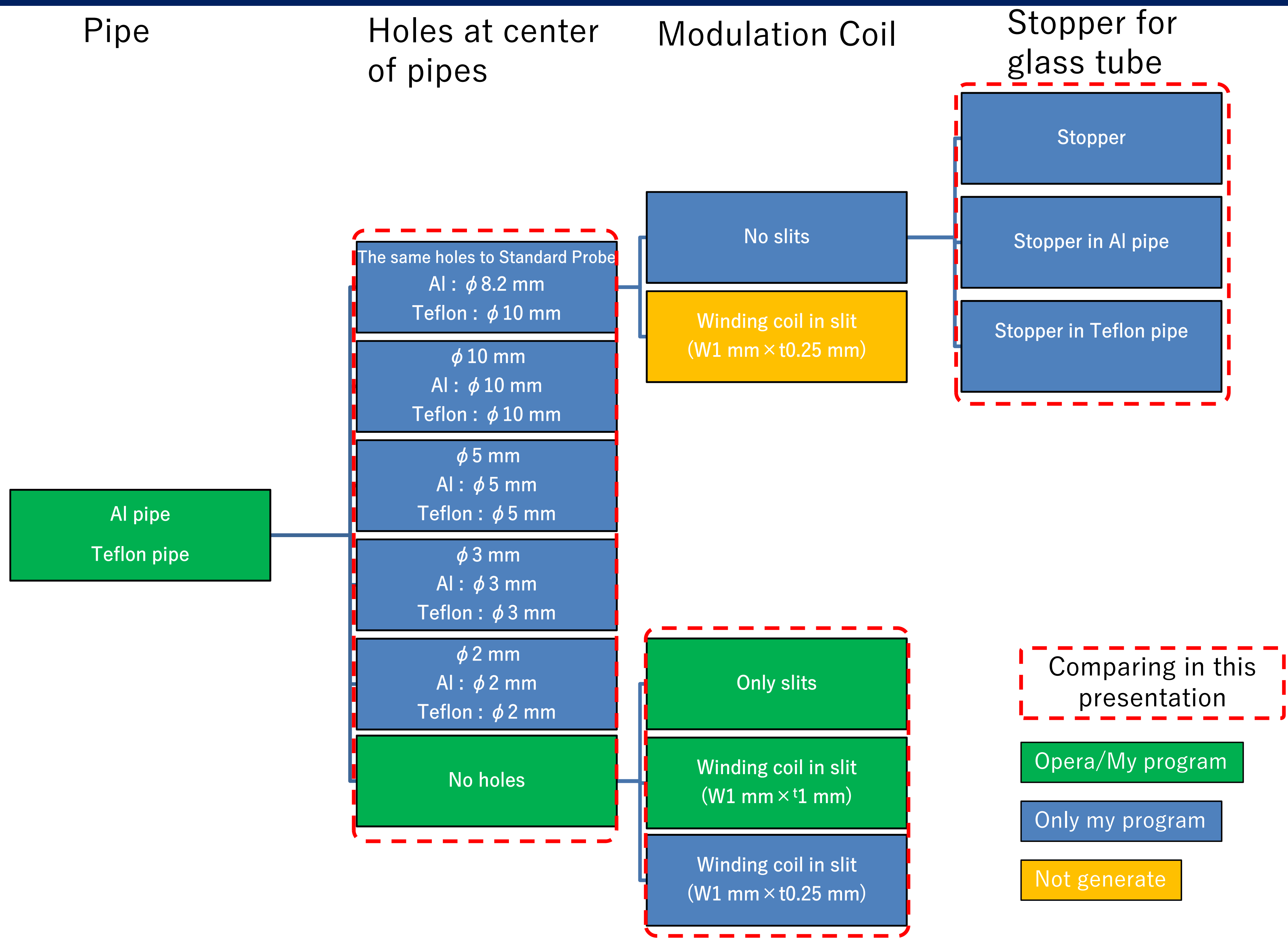
$$\phi = \frac{q_m}{4\pi\mu_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \sim \frac{q_m}{4\pi\mu_0}\frac{1}{r}\frac{\ell\cos\theta}{r} = \frac{M}{4\pi\mu_0 r^2}\cos\theta$$

$$\Rightarrow \begin{cases} H_r = -\dfrac{\partial\phi}{\partial r} = \dfrac{M}{2\pi\mu_0 r^3}\cos\theta \\ H_\theta = -\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta} = \dfrac{M}{4\pi\mu_0 r^3}\sin\theta \end{cases}$$



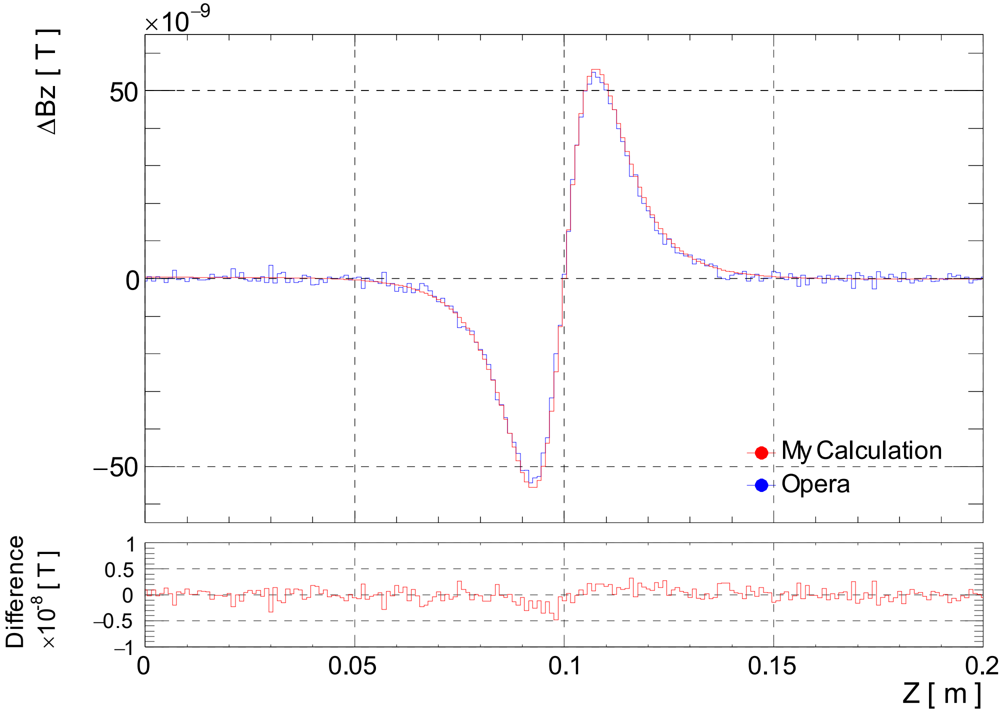
In the material, mesh size (ℓ) is not large than r

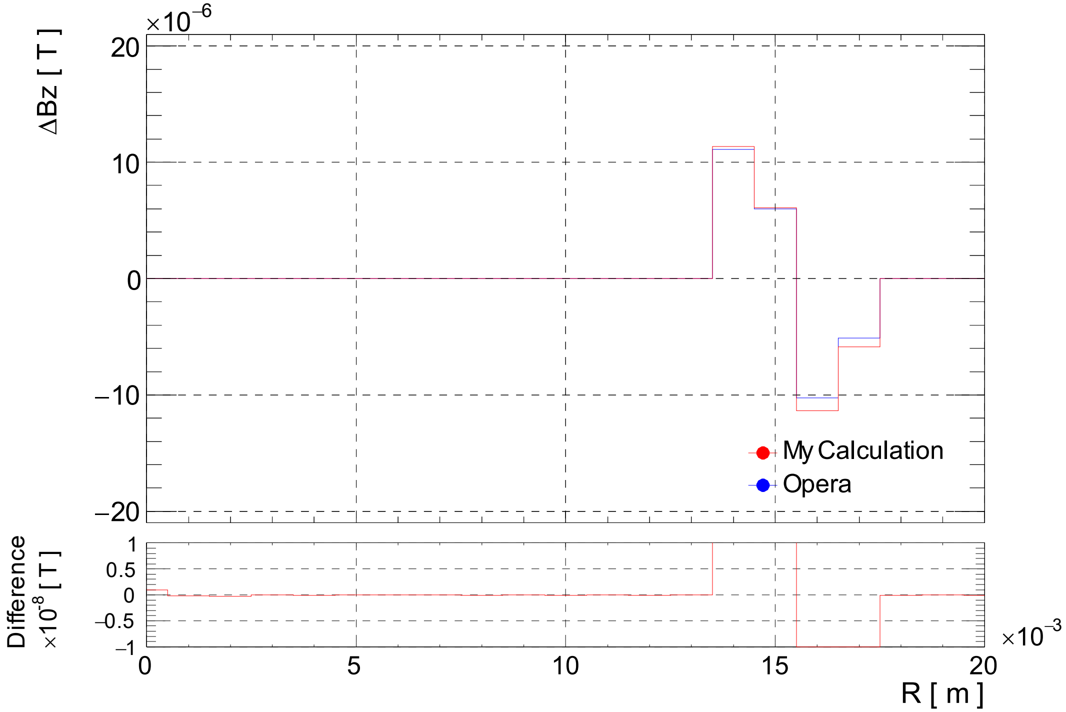
Dataset


Pipe
Holes at center of pipes
Modulation Coil
Stopper for glass tube
Al pipe
Teflon pipe
The same holes to Standard Probe
Al : ϕ8.2 mm
Teflon : ϕ10 mm
ϕ10 mm
Al : ϕ10 mm
Teflon : ϕ10 mm
ϕ5 mm
Al : ϕ5 mm
Teflon : ϕ5 mm
ϕ3 mm
Al : ϕ3 mm
Teflon : ϕ3 mm
ϕ2 mm
Al : ϕ2 mm
Teflon : ϕ2 mm
No holes
No slits
Winding coil in slit
(W1 mm×t0.25 mm)
Stopper
Stopper in Al pipe
Stopper in Teflon pipe
Only slits
Winding coil in slit
(W1 mm×t1 mm)
Winding coil in slit
(W1 mm×t0.25 mm)
Comparing in this presentation
Opera/My program
Only my program
Not generate

Comparison with Opera

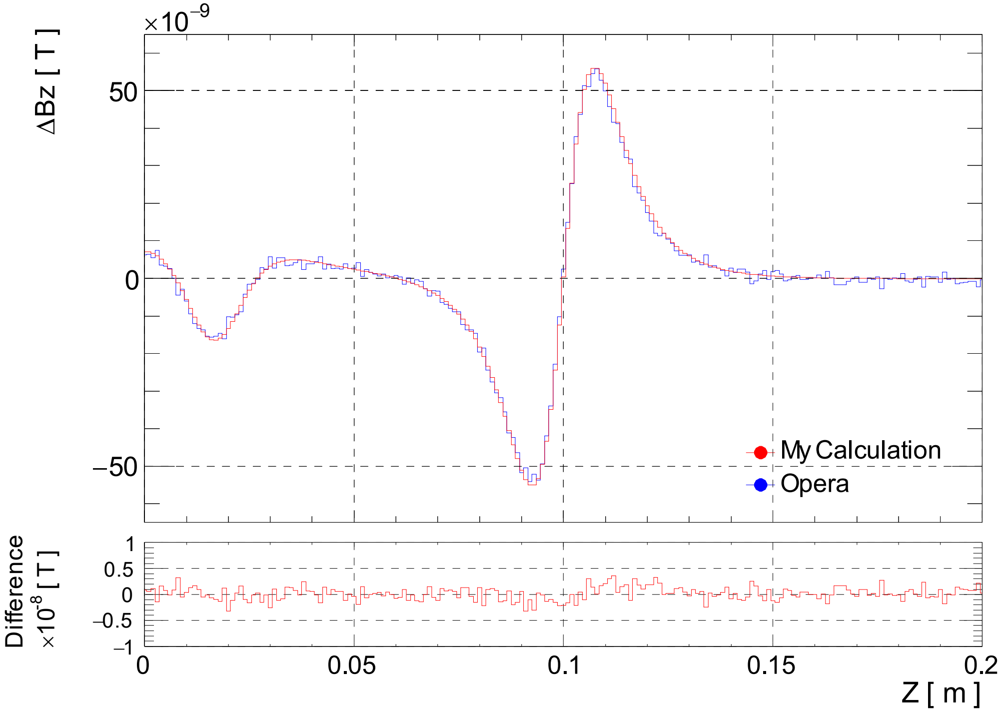
Al and Teflon pipes without holes and slits

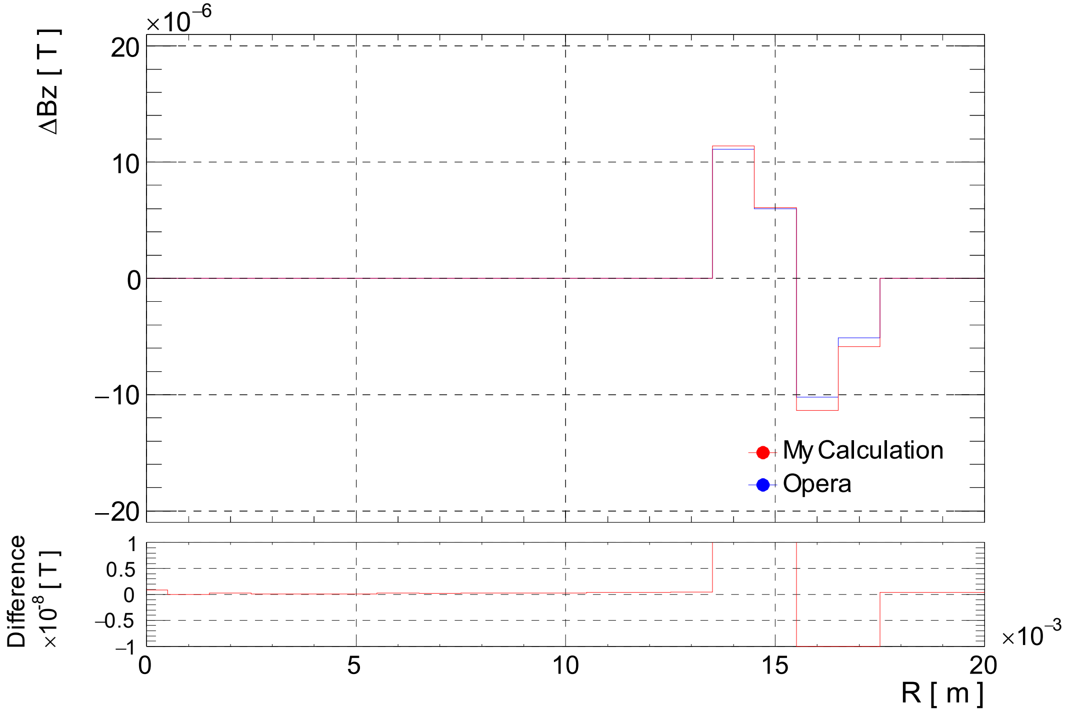




Comparison with Opera

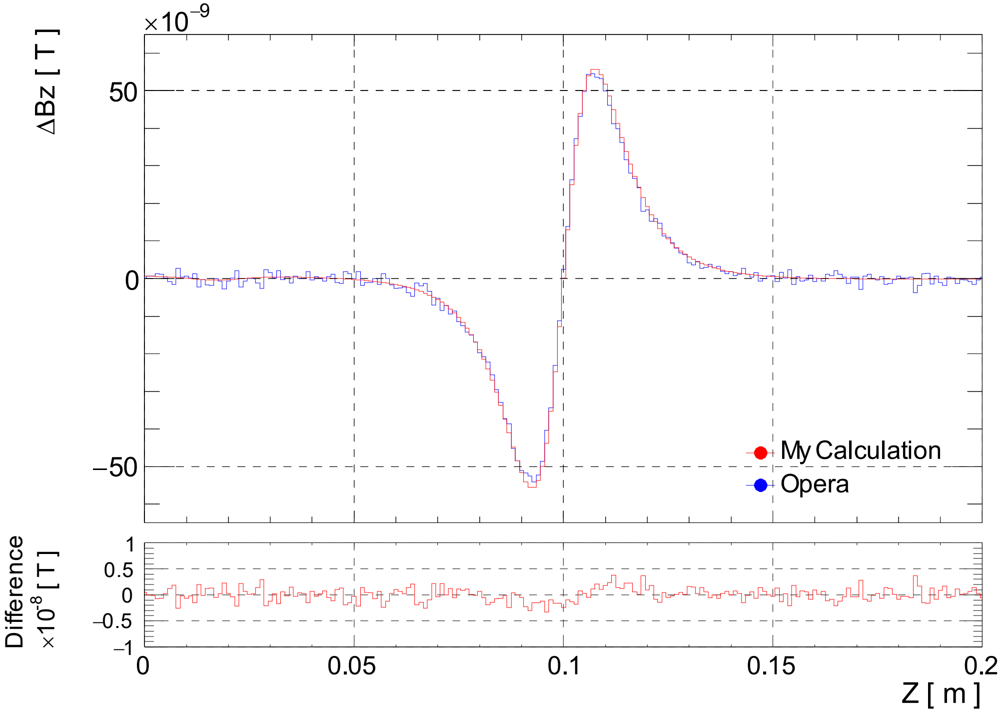
Al and Teflon pipes with holes and slits

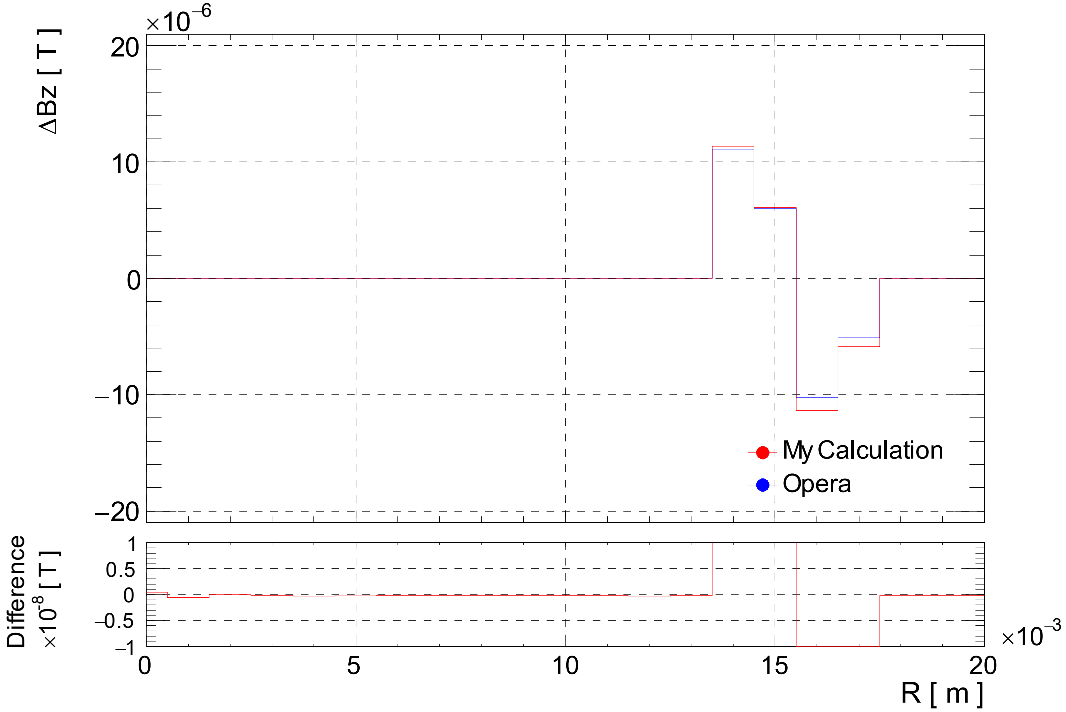




Comparison with Opera

Al and Teflon pipes with holes and slits and the modulation coils are winded

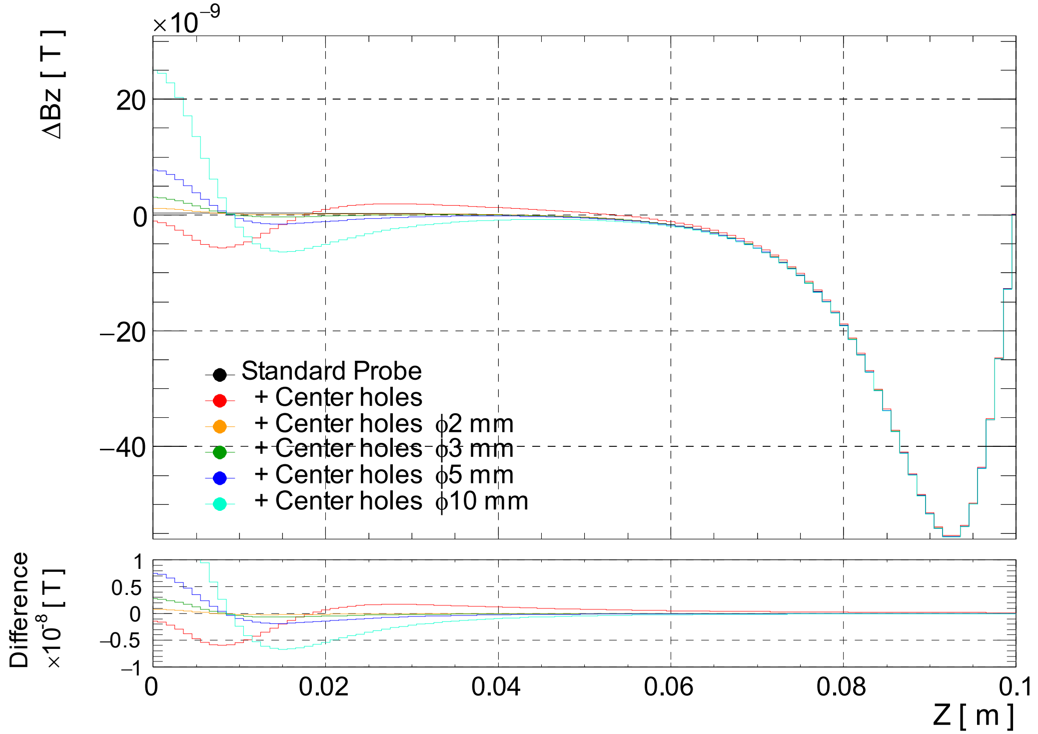


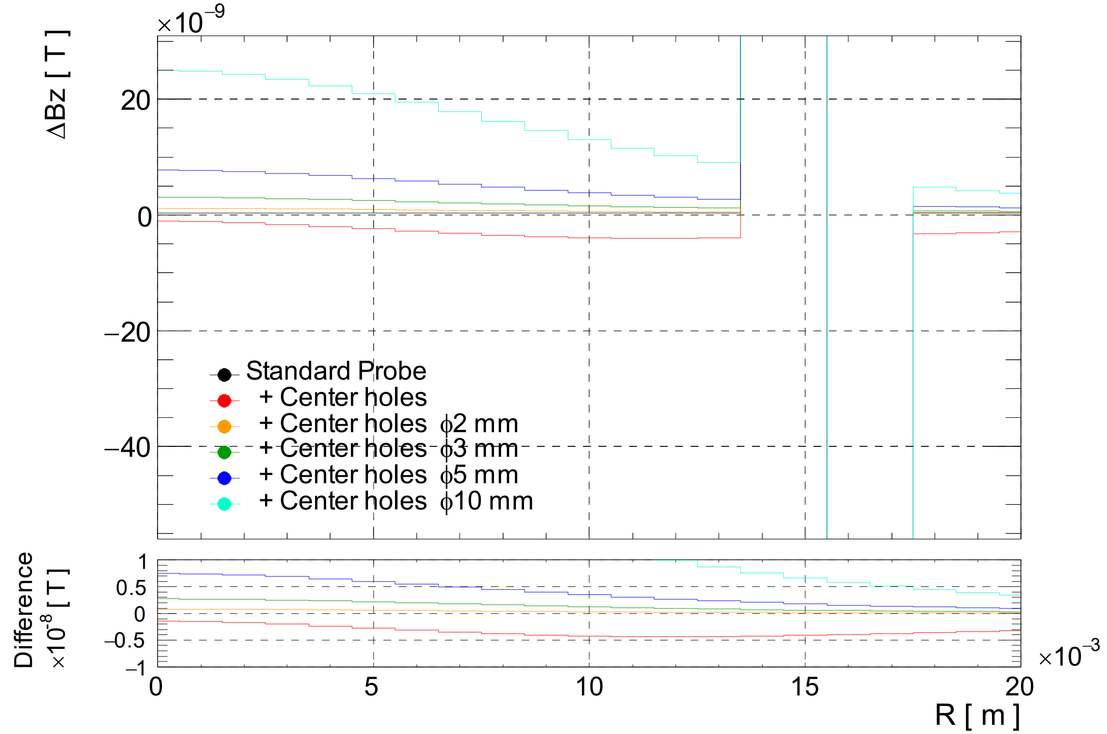


The agreement is less than 5 ppb (except the regions in materials)

Diameters of Holes

Comparison Magnetic fields in different hole size



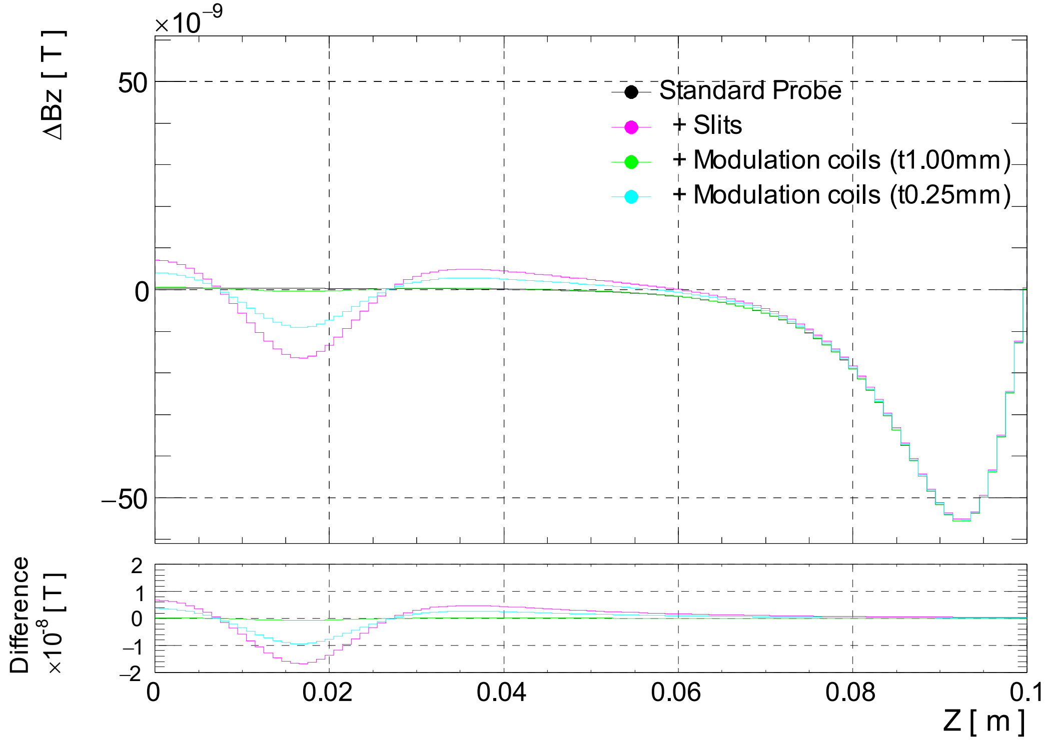


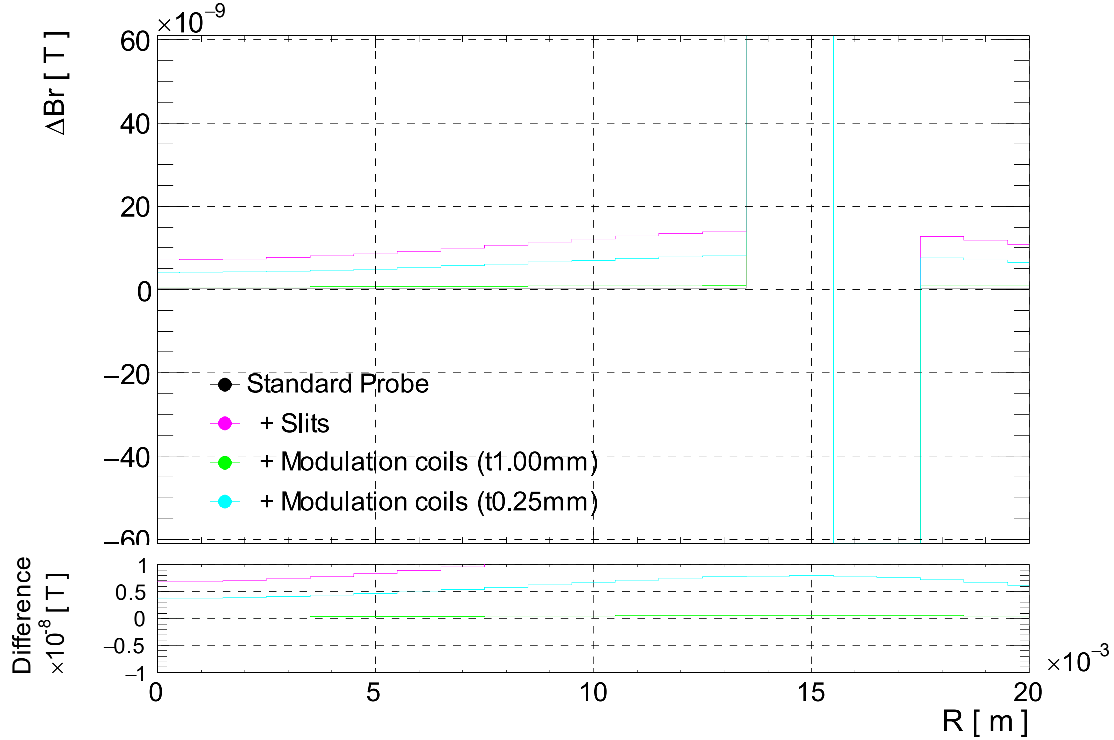
Bottom plots show difference of error field from Al and Teflon pipe without holes and slits

The smaller hole size become smaller error field

Slits for Modulation Coil

Effect derived from slits for modulation coils
(The center holes are not made in this calculation)





The susceptibility of Teflon is approximate to that of copper

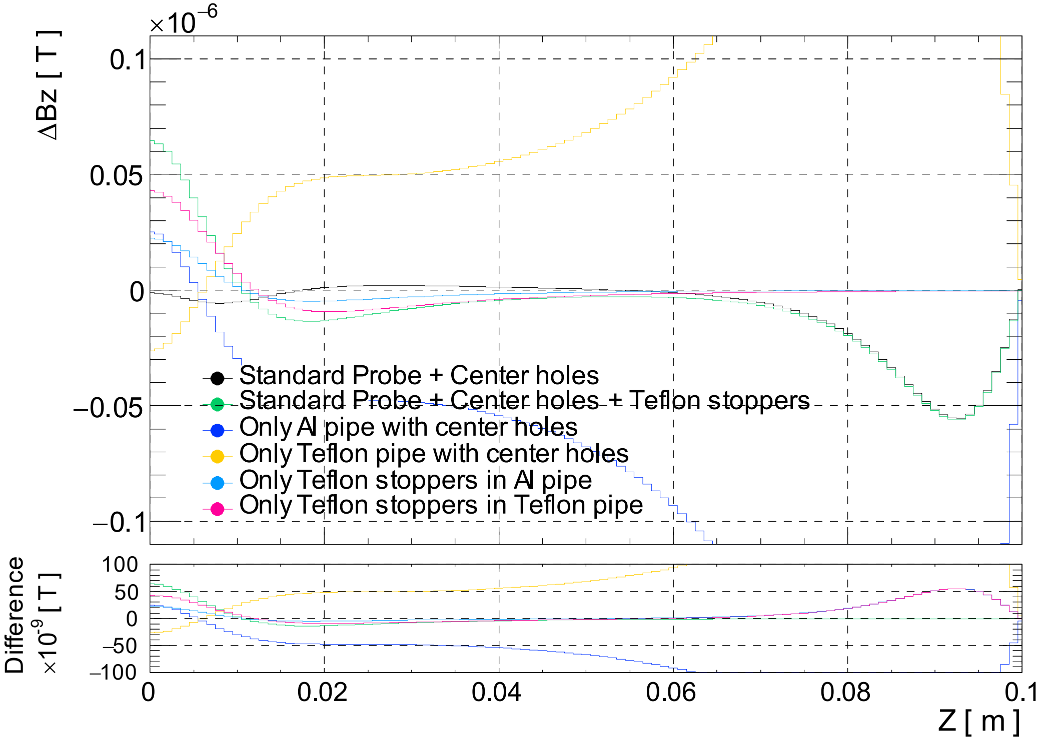
Teflon : $\chi = -1.025 \times 10^{-5}$

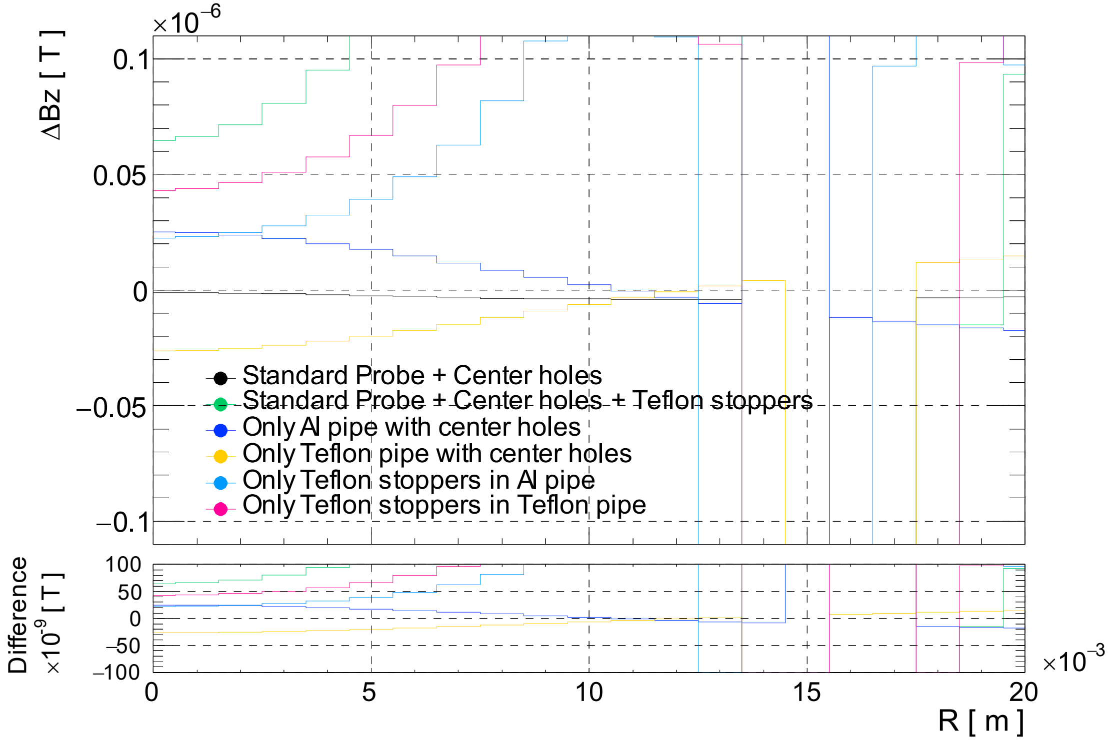
Cu : $\chi = -9.80 \times 10^{-6}$

The error field is negligible when the slit for modulation coil is filled with copper wires

Stoppers

Teflon stoppers inserted into the center holes are included in my calculation program





Effects derived from stoppers are not cancelled because the region in Al pipe layer is filled with Teflon

Error field in Standard Probe

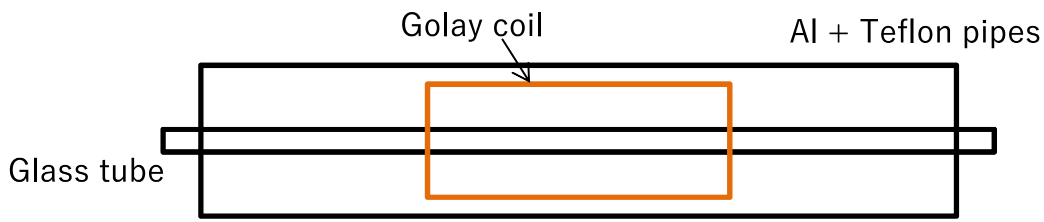
	Error Field at Center of Probe
Just Al pipe and Teflon pipe	+0.3 ppb
Center holes	-1.0 ppb
Slits (without modulation coil)	+7.1 ppb
Modulation coils	+4.1 ppb
Stopper	+64.6 ppb

The error field derived from stopper is much large

Next Standard Probe

- Smaller holes and stoppers, shallow slits as possible

- Glass tube is set in parallel direction with pipes
  - Golay coils is used in this case



Cross Calibration in March

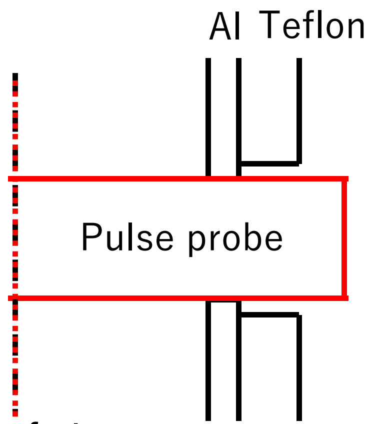
Material effect

	Measurement [ppb]	Calculation [ppb]
Al pipe	+32 ± 3	+26.7
Teflon pipe	-75 ± 3	-27.7
Al + Teflon pipe	-14 ± 1	-1.0

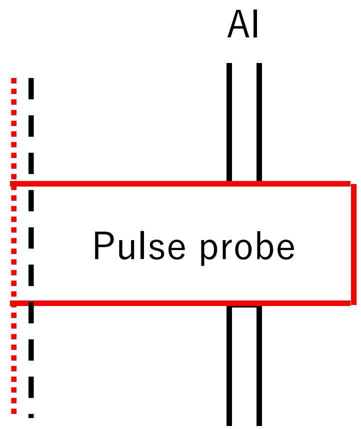
"Al pipe" + "Teflon pipe" should be same to "Al+Teflon pipe"

Conceivable source of this disagreement is position misalignment

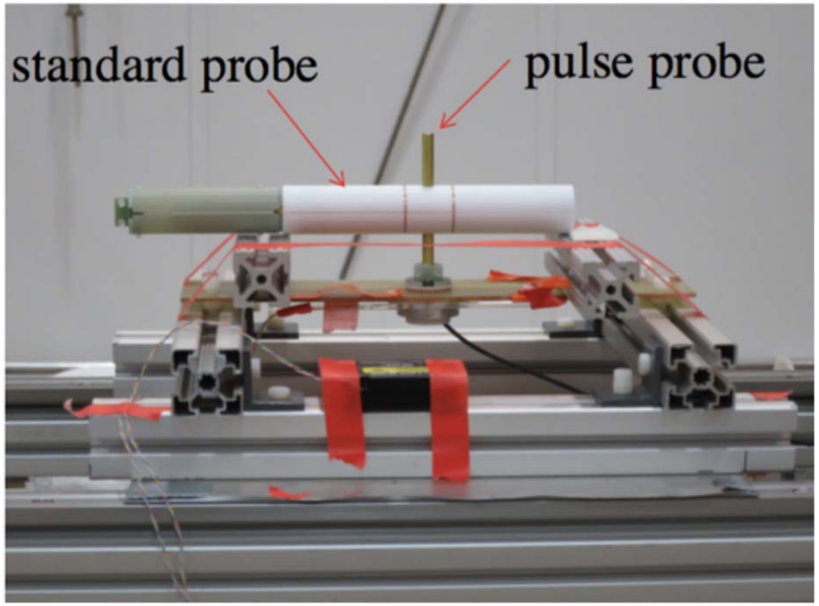
At Al+Teflon measurement



At Al measurement



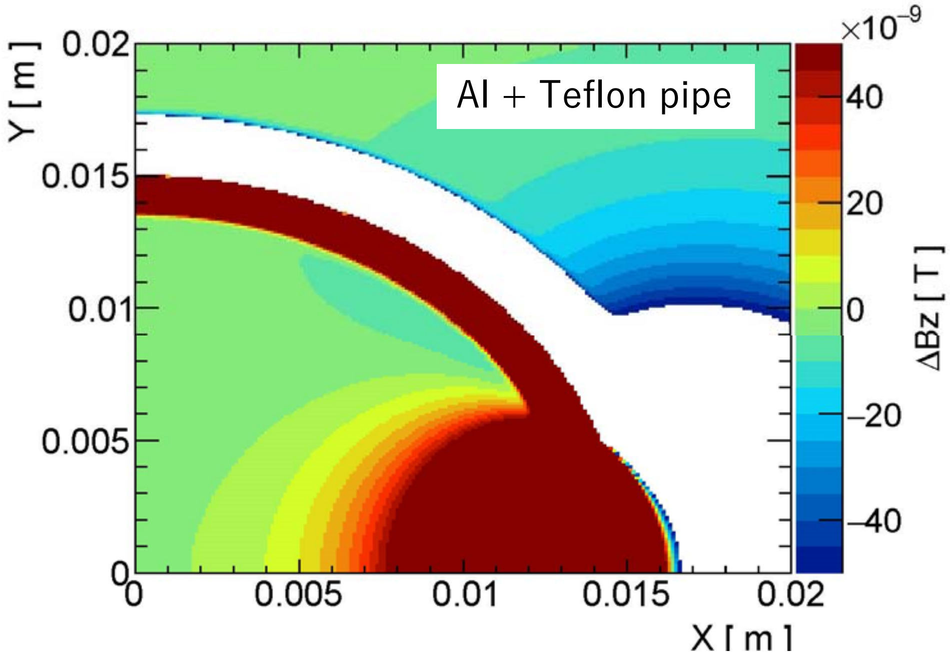
Center of pipes
Center of pulse probe

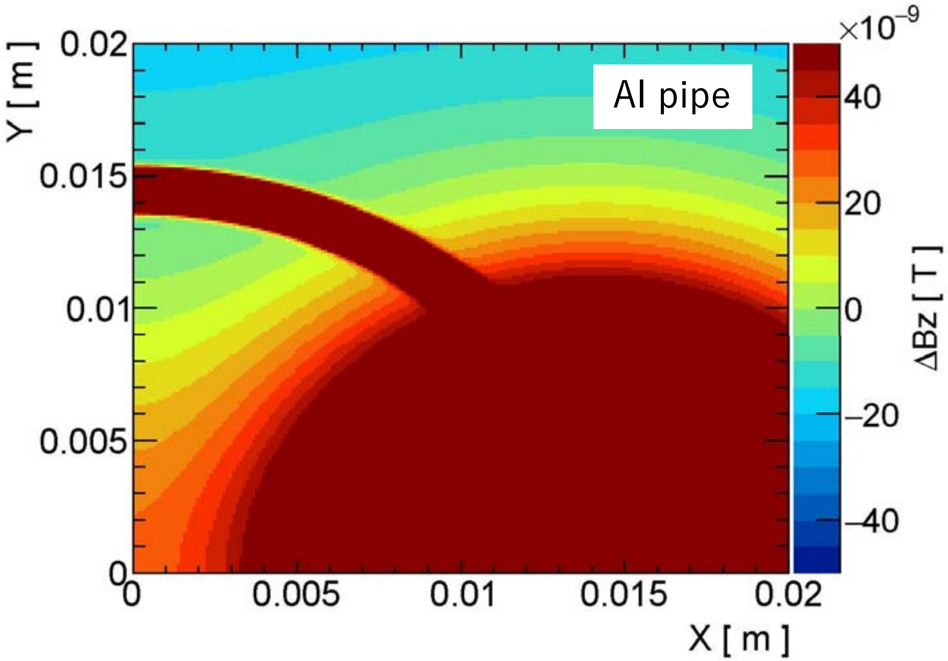


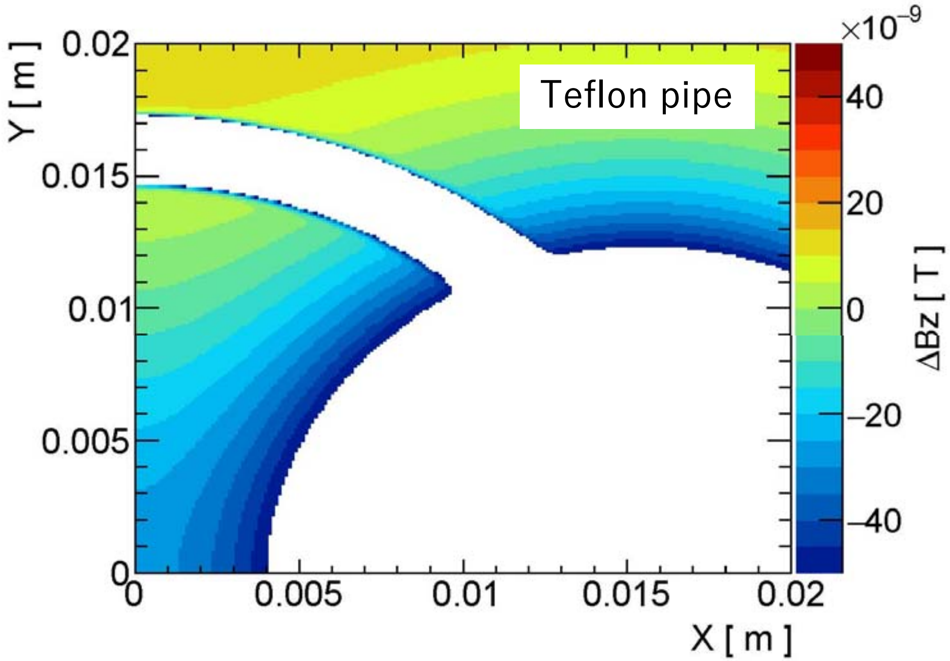
Estimate difference of flux when the position of probe is moved up to 3 mm
(misalignment : 2 mm/position uncertainty : 1 mm)

Flux Map

Pulse tube was inserted along X axis







Material Effects

	Measurement	Calculation Corresponding position	Calculation	
			X = 0 mm	X = 3 mm
Al pipe	+32 ± 3 ppb	(X = 1.8 mm)	+26.7 ppb	41.8 ppb
Teflon pipe	-75 ± 3 ppb	(X = 5.6 mm)	-27.7 ppb	-39.2 ppb
Al + Teflon pipe	-14 ± 1 ppb	(X = 5.5 mm)	-1.0 ppb	2.1 ppb

"Corresponding position" is the position where strength of flux on my calculation is the same to that of measurement

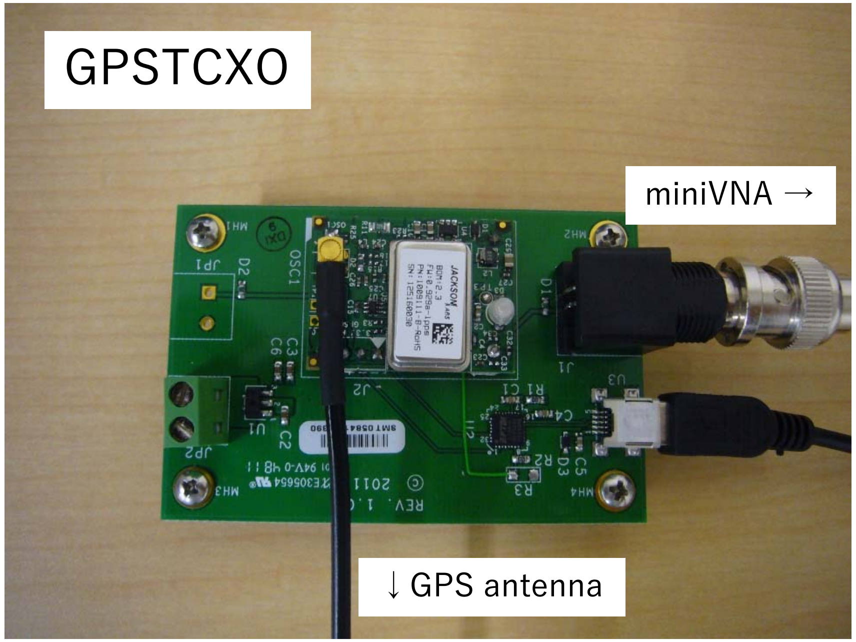
Measurement of "Al pipe" is reasonable agreement with calculation
On the other hand, "Teflon pipe" and "Al + Teflon pipe" are not reconstructed by my calculation

- The disagreement is impossible to be described by misalignment or position uncertainty
- We have not understood the source of disagreement

Preparations for Next Cross Calibration

Sasaki-san has been already choosing and purchasing some devices to utilize for next cross calibration

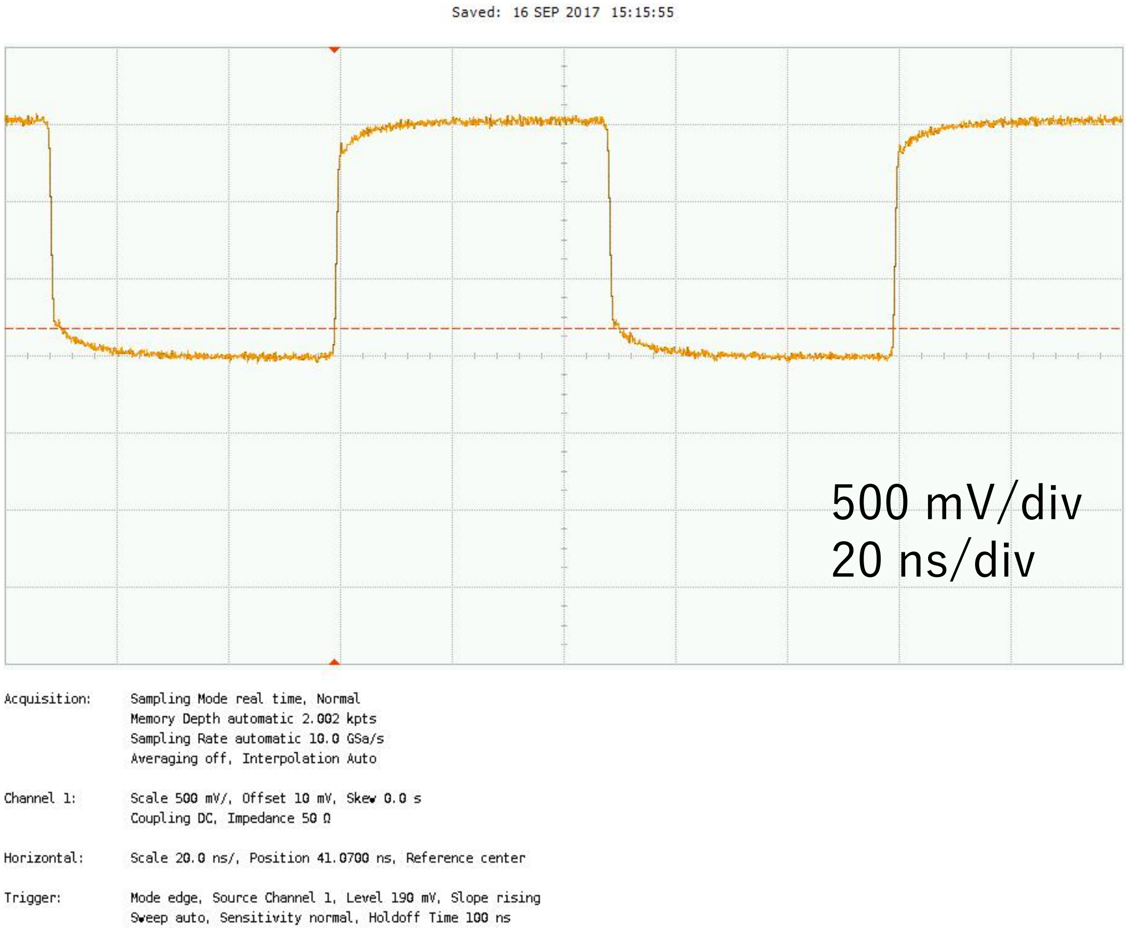
- Precious 10 MHz generator synchronized with GPS : GPSTCXO
- Frequency analyzer : miniVNA Pro
- Ultrasonic motors for stepping stage

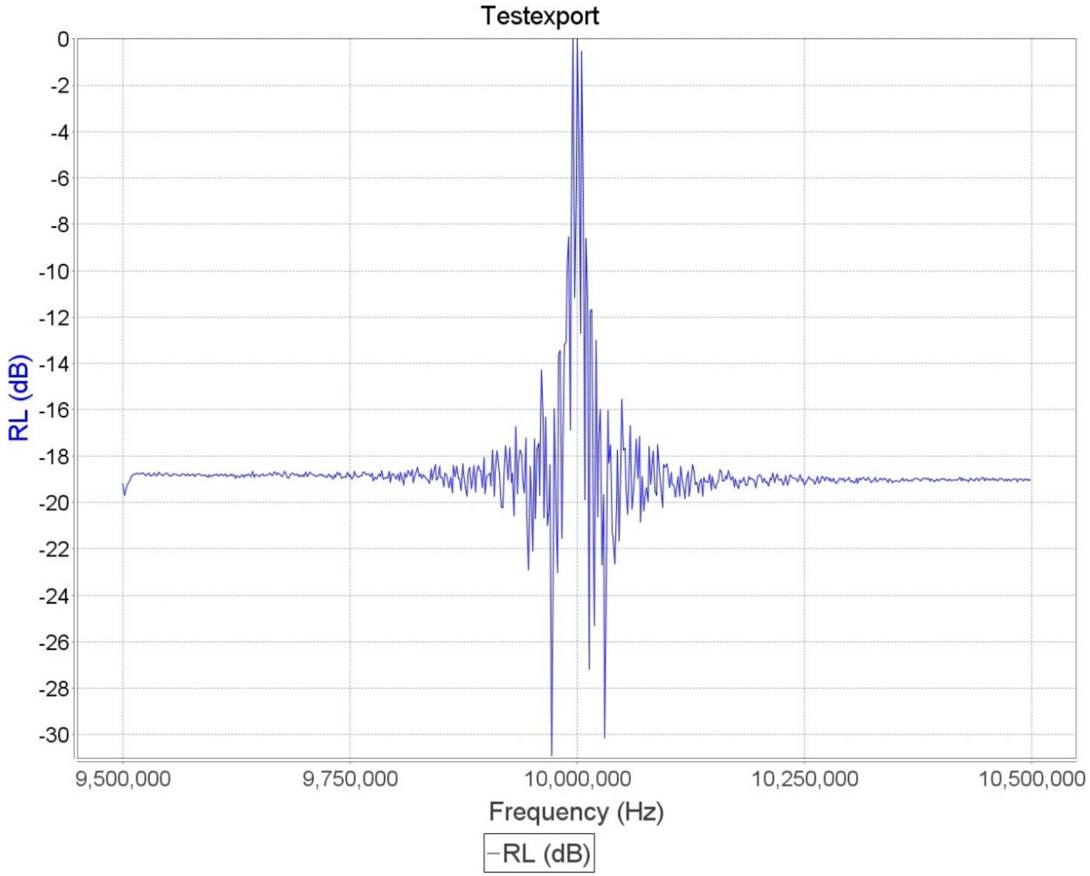


Output 10 MHz (100 nsec)

Preparations for Next Cross Calibration

GPSTCXO output (read by oscilloscope)





miniNVAPro

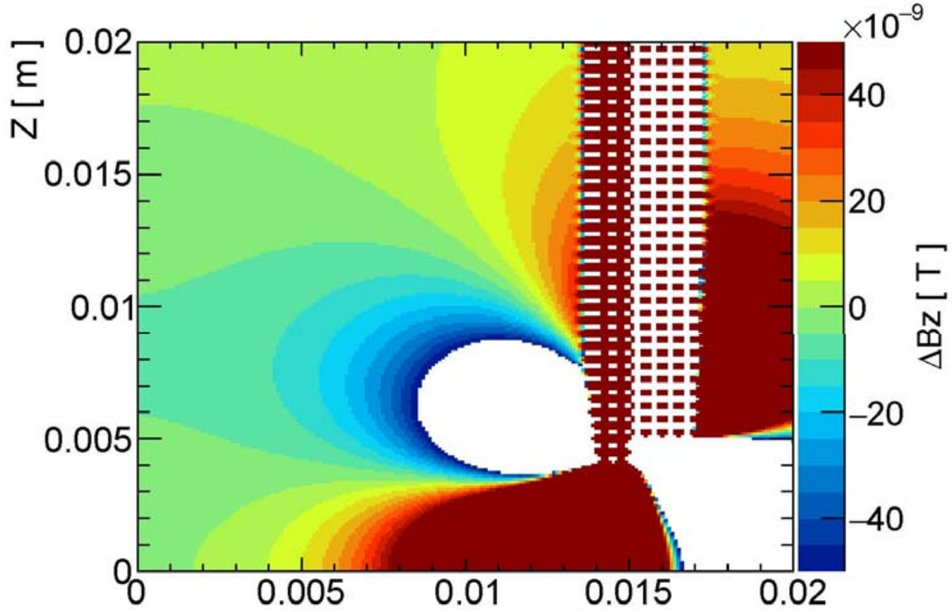
Summary

- Calculate the magnetic field to improve next cross calibration test
  - Contributions from holes and stopper are much larger

- Measurement of material effect is not consistent with calculation

- Some devices are ready for next cross calibration

Backup


Z [m]
0.02
0.015
0.01
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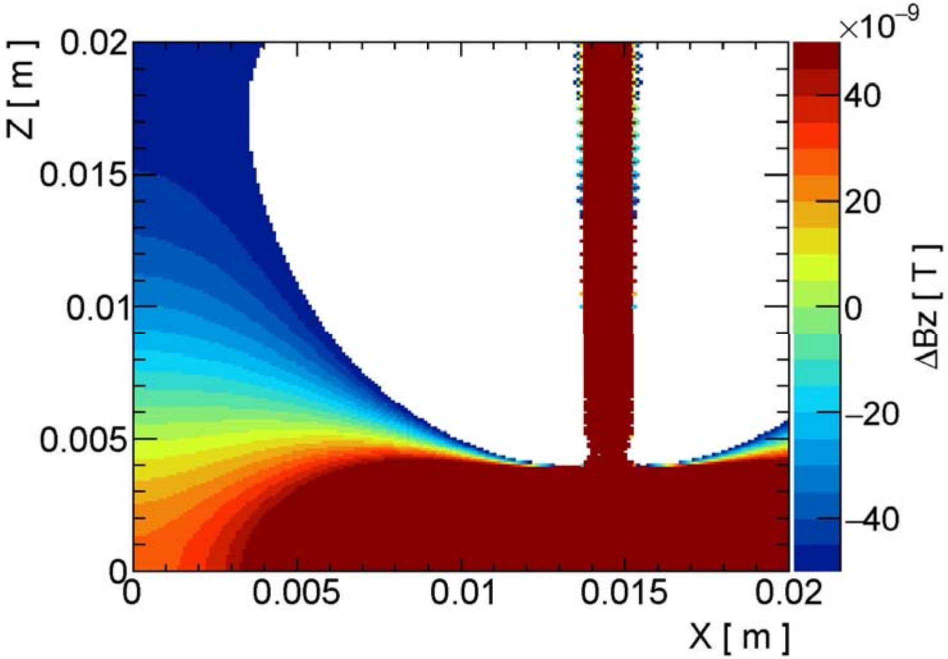
ΔBz [T]


Z [m]
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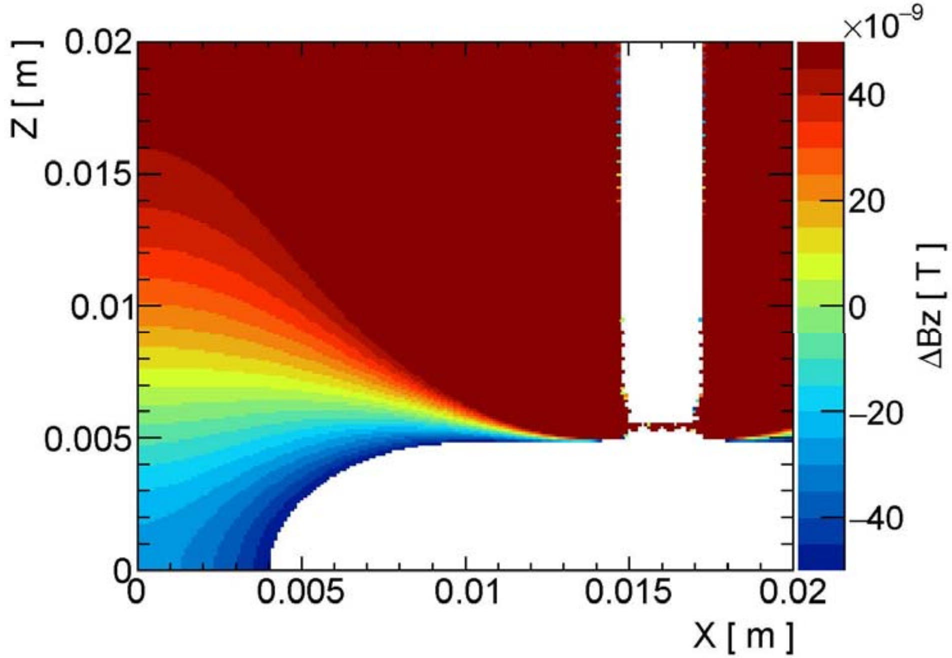
Z [m]
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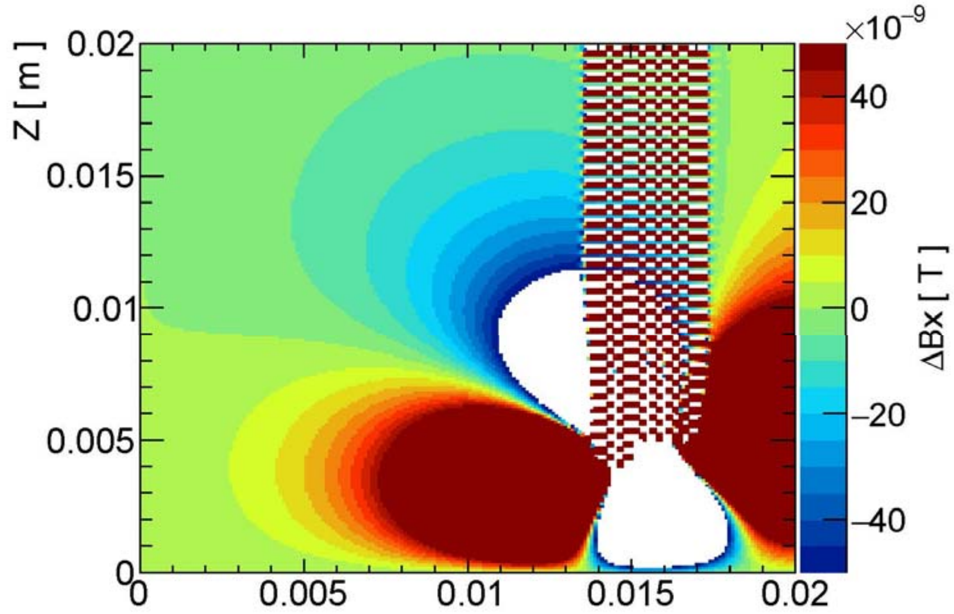
ΔBz [T]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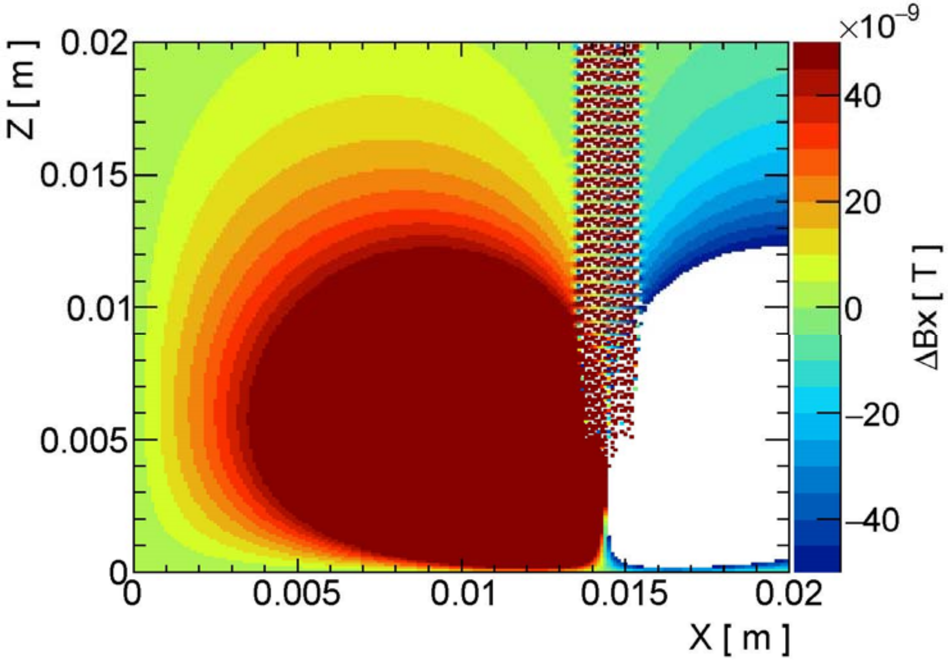
ΔBx [T]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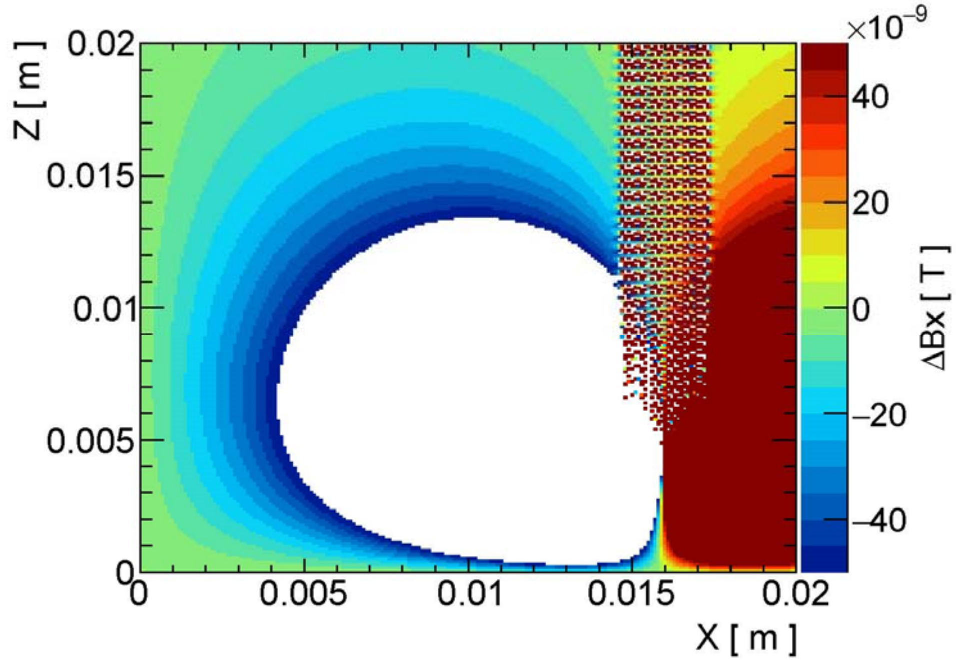
ΔBx [T]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X [m]
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ΔBx [T]