Dark Odyssey 2020: Gravitational-Wave Probes of Dark Universe,
Jan 4-7, 2020, SNU, Korea

Primordial Black Hole and Stochastic Gravitational Wave Induced by Scalar Perturbations

Rong-Gen Cai

Institute of Theoretical Physics
Chinese Academy of Sciences

RGC, S. Pi and M. Sasaki, arXiv: 1810.11000; PRL 122, 201101 (2019）
RGC, S. Pi, S.J. Wang and X.Y. Yang, arXiv. 1901.10152; JCAP05, 013 (2019)
RGC, S. Pi, S.J. Wang and X.Y. Yang, arXiv: 1907.06372; JCAP 1910, 059 (2019)
RGC, S.Pi and M. Sasaki, arXiv: 1909.13728
RGC, Z.K. Guo, J. Liu, L. Liu and X.Y. Yang, arXiv: 1912.10437

Outline:

1) Introduction

2) GWs induced scalar perturbations

3) Resonant multiple peaks in the induced GWs

4) PTA Constraints on the induced gravitational waves

5) PBH and GWs from parametric amplification of
 curvature fluctuations

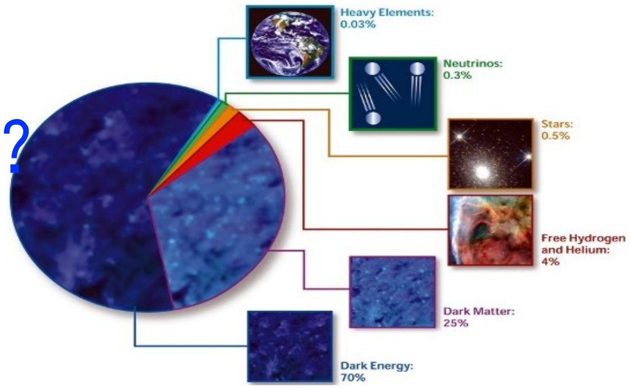
Standard cosmological model:

inflation $\oplus$ big bang $\oplus$ dark matter $\oplus$ dark energy

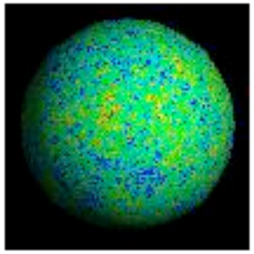
5% 28% 67%

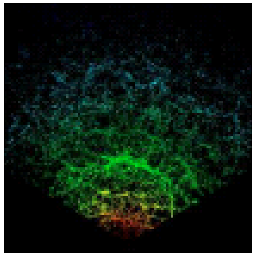
Challenges:

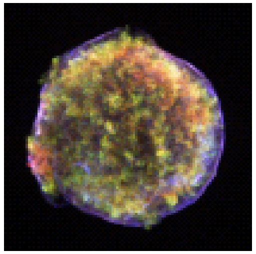
inflation ? dark matter ?
dark energy? general relativity ?

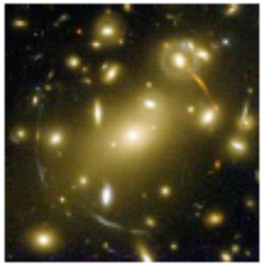


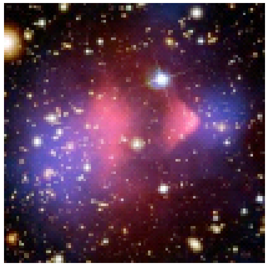
Probes:


Microwave background


Galaxy surveys


Supernovae Ia


Gravitational lensing


Galaxy clusters

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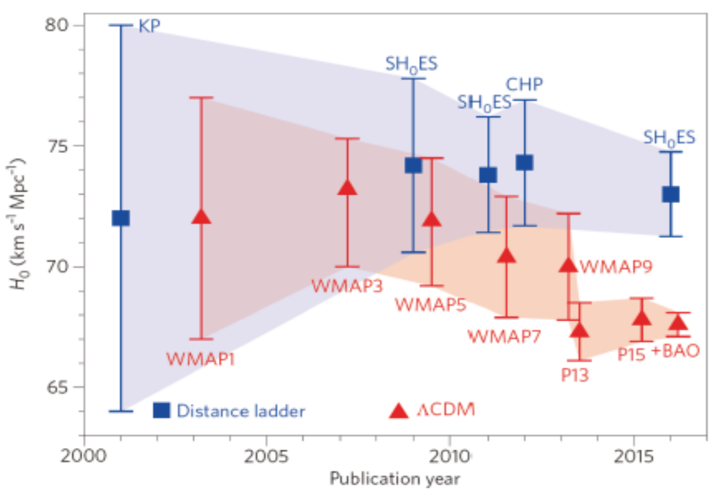
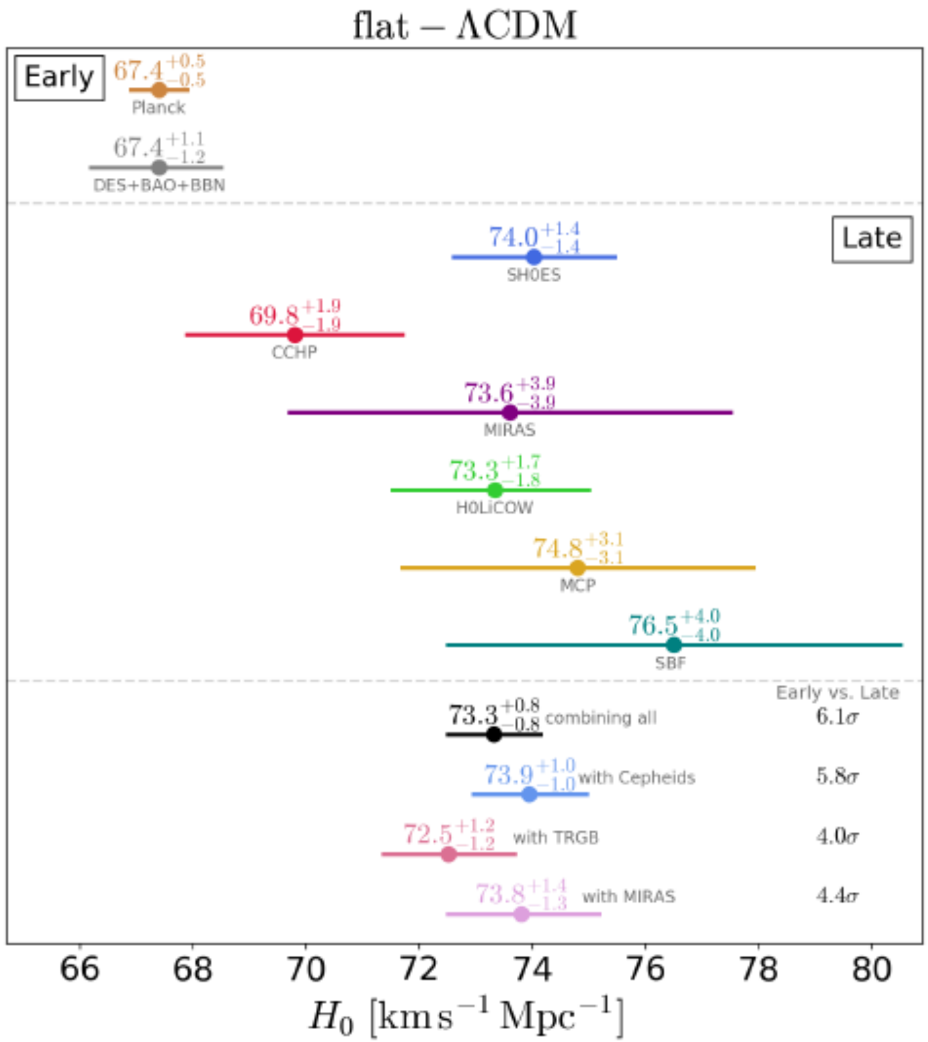
LCDM model

ΛCDM model:

1907.10625

◆ H_0 tension?

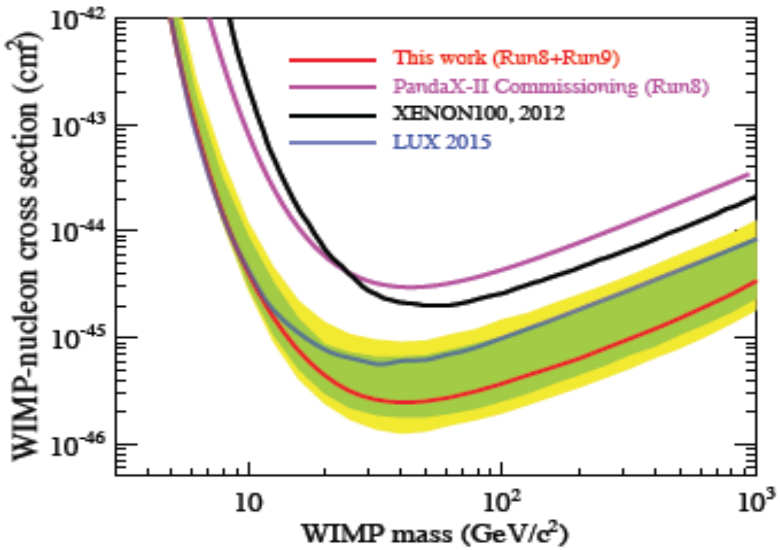




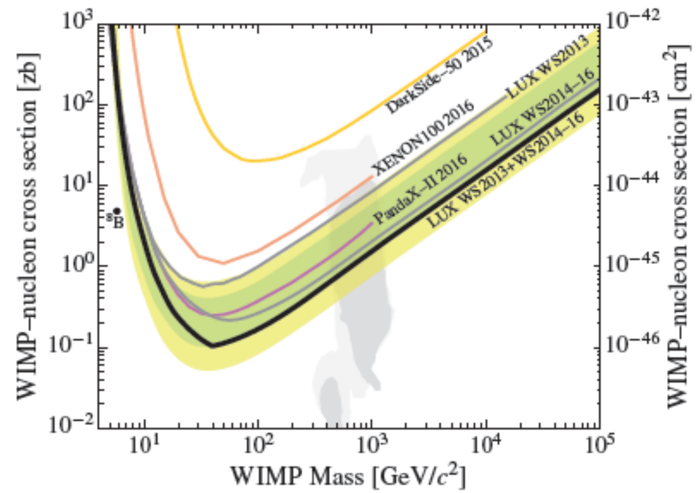
◆ sigma_8

ΛCDM model:

◆ WIMP?



PandaX−II, PRL 117 (2016)121303



LUX, arXiv:1608.07648

◆ Collider & Non-indirect ?

[11] van Dokkum P. et al. A galaxy lacking dark matter. *Natur.* **555**, 629-632 (2018).

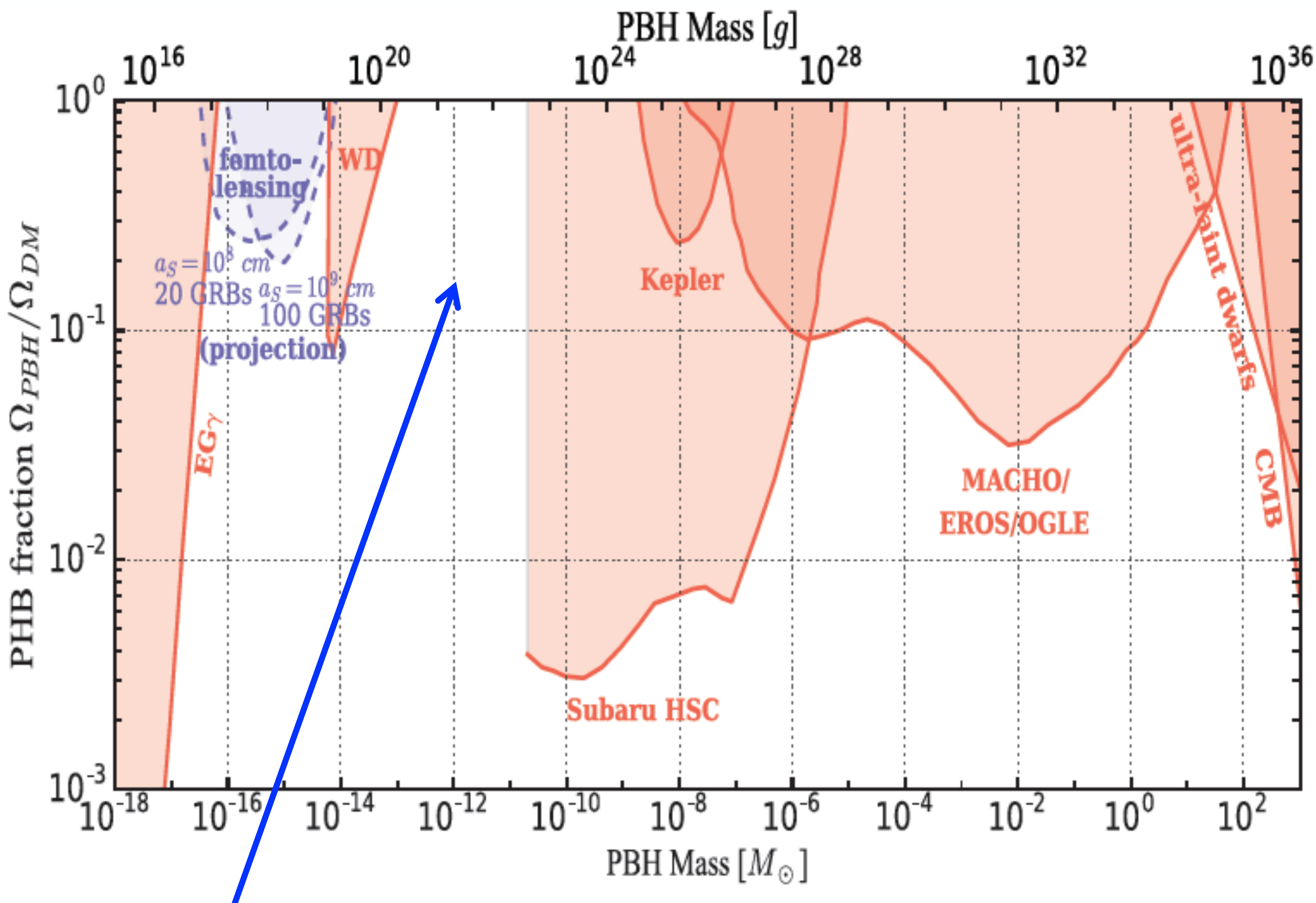
arXiv: 1908.00046, Nature astronomy, Nov, 2019

A population of dwarf galaxies deficient in dark matter

Qi Guo*[1,2,3]; Huijie Hu[1,2];Zheng Zheng[1,4]; ShiHong Liao[1,3]; Wei Du[1]; Shude Mao[1,5,6]; Linhua Jiang[7]; Jing Wang[7]; Yingjie Peng[7]; Liang Gao[1,2,3]; Jie Wang[1,2,3]; Hong Wu[1,2]

faintness of such systems in the past. Here we report 19 dwarf galaxies that could mainly consist of baryons up to radii well beyond r_e, where they are expected to be dominated by dark matter. 14 of them are isolated dwarf galaxies, free from the influence of nearby bright galaxies/AGNs and high dense environments. This result provides observational evidence that could challenge the formation theory of low-mass galaxies in the framework of standard cosmology.

Primordial black hole as a candidate of dark matter?



Inflation model and primordial gravitational wave?

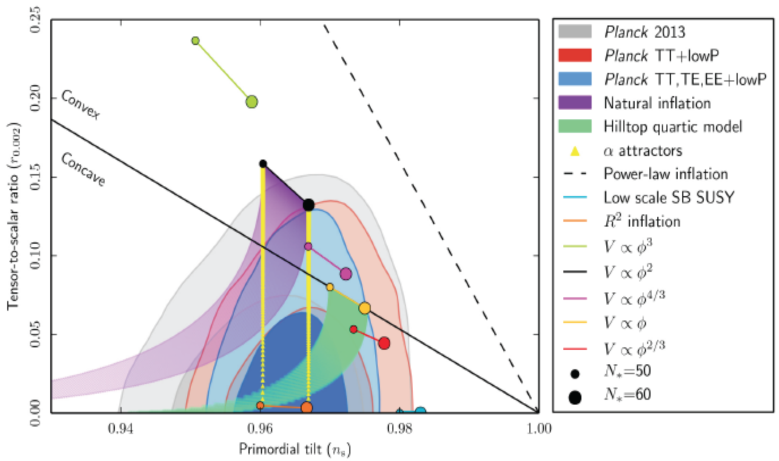
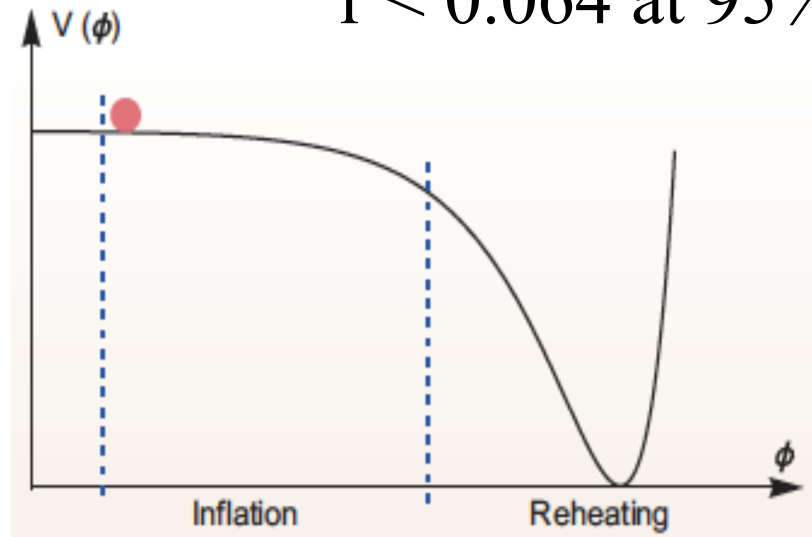
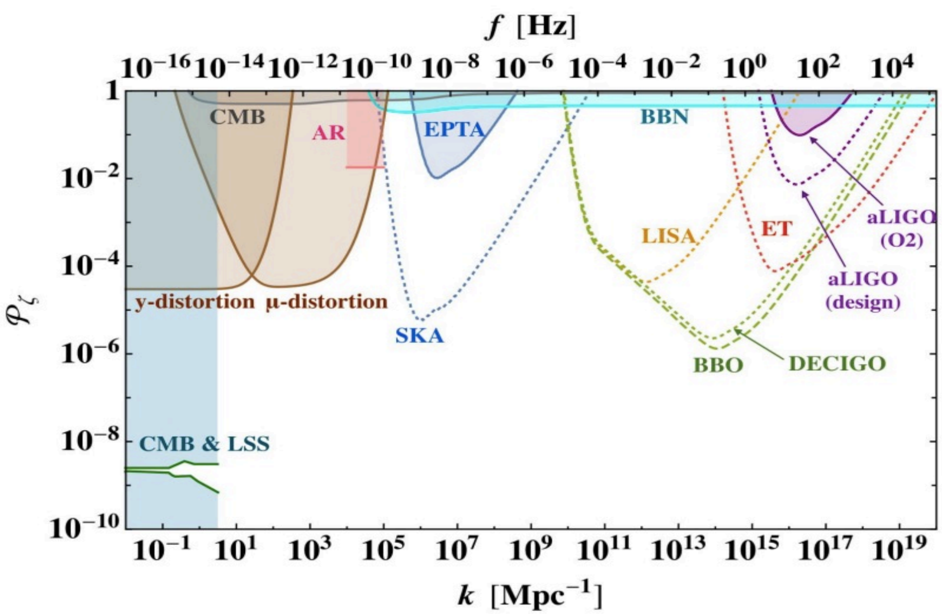


Figure 2: Marginalized joint 68% and 95% confidence level regions for n_s and $r_{0.002}$ from the Planck 2015 data, compared to the theoretical predictions of some inflationary models. This figure is quoted from [11].

r < 0.064 at 95%!





✧ What and When?

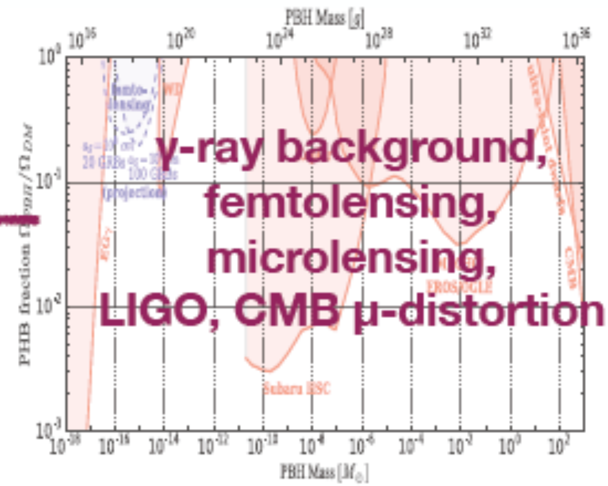
✧ Perturbations at small scales?

✧ TCC? r <6.8 x 10^(-33)
arXiv: 1909.11106

Inflation:

Induced GWs

Peak Theory

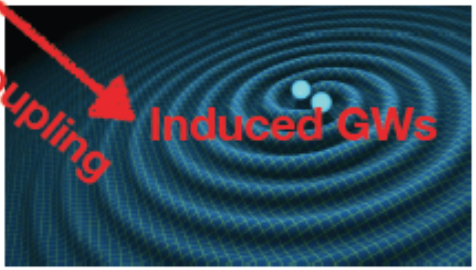


γ-ray background, femtolensing, microlensing, LIGO, CMB μ-distortion

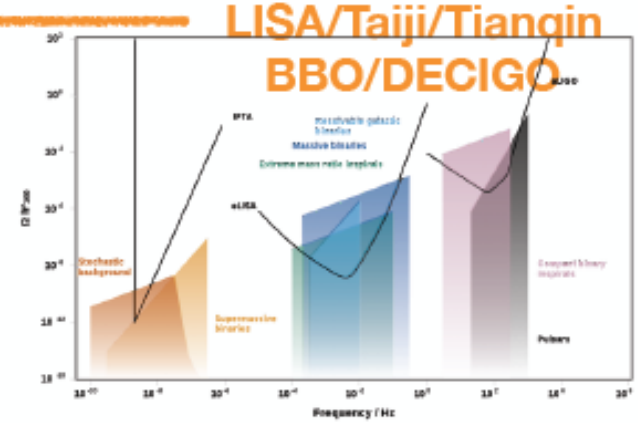
?

Peak of scalar perturbation on small scales

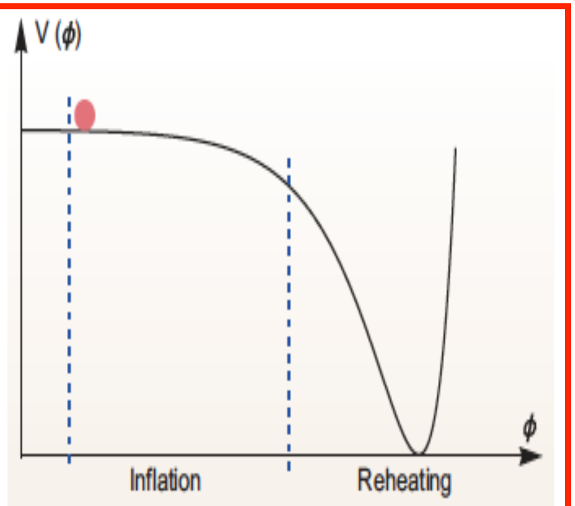
secondary coupling



LIGO/VIRGO/KAGRA LISA/Taiji/Tianqin BBO/DECIGO



?



2) Gravitational waves induced by scalar perturbations, R.G. Cai, S. Pi and M. Sasaki, arXiv: 1810.11000

The perturbed metric in the Newton gauge:

$$ds^2 = a^2[-(1-2\Phi)\,d\eta^2 + ((1+2\Phi)\delta_{ij} + h_{ij})dx^i dx^j] \,.$$

The equation of motion of tensor perturbation in the radiation dominated era:

$$\begin{aligned} & h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} \\ = \ & 18 \int \frac{d^3 l}{(2\pi)^{3/2}} \frac{l^2}{\sqrt{2}} \sin^2\theta \begin{pmatrix} \cos 2\varphi \\ \sin 2\varphi \end{pmatrix} \Phi_{\mathbf{l}} \Phi_{\mathbf{k}-\mathbf{l}} \\ & \times \left[j_0(ux) j_0(vx) - 2\frac{j_1(ux) j_0(vx)}{ux} \right. \\ & \left. -2\frac{j_0(ux) j_1(vx)}{vx} + 6\frac{j_1(ux) j_1(vx)}{uvx^2} \right] . \end{aligned}$$

$$u = |\mathbf{k}-\mathbf{l}|/k, \; v = l/k$$

$$x = k\eta/\sqrt{3}.$$

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After solving $h_{\mathbf{k}}$, one can obtain the density parameter $\Omega_{\rm GW}(k)$ defined as the energy density of the GW per unit logarithmic frequency normalized by the critical density

$$\Omega_{\rm GW}(k) \equiv \frac{1}{12}\left(\frac{k}{Ha}\right)^2 \frac{k^3}{\pi^2}\overline{\langle h_{\mathbf{k}}(\eta)h_{\mathbf{k}}(\eta)\rangle},$$

where the overline means the time average.

The curvature perturbation in the comoving sclices can be expressed in terms of the Gaussian part as

$$\mathcal{R}(\mathrm{x}) = \mathcal{R}_g(\mathrm{x}) + F_{\rm NL}\left[\mathcal{R}_g^2(\mathrm{x}) - \langle\mathcal{R}_g^2(\mathrm{x})\rangle\right], \qquad \Phi = (2/3)\mathcal{R}$$

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the non-linear parameter

Then the two-point correlation function of $\Phi_{\mathbf{k}}$

$$\langle \Phi_{\mathbf{k}} \Phi_{\mathbf{p}} \rangle \sim \frac{4}{9} \left(P_{\mathcal{R}}(k) + 2F_{\rm NL}^2 \int d^3 l \; P_{\mathcal{R}}(|\mathbf{k}-\mathbf{l}|) P_{\mathcal{R}}(l) \right)$$

where we omitted an overall factor $(2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{p})$, and the Gaussian power spectrum is defined as $\langle \mathcal{R}_{g,\mathbf{k}} \mathcal{R}_{g,\mathbf{p}} \rangle = (2\pi)^3 P_{\mathcal{R}}(k) \delta^{(3)}(\mathbf{k}+\mathbf{p})$.

Now we consider a primordial curvature perturbation with a narrow peak at some specific scale k_*,

$$P_{\mathcal{R}}(k) = \frac{\mathcal{A}_{\mathcal{R}}}{(2\pi)^{3/2} 2\sigma k_*^2} \exp\left(-\frac{(k-k_*)^2}{2\sigma^2} \right).$$

here the width $\sigma \ll k_*$;

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The power spectrum of the tensor perturbation, up to the epoch of radiation-matter equality:

$$\Omega_{\rm GW} = 6\mathcal{A}_{\mathcal{R}}^2 \frac{k^2}{2\pi\sigma^2}\left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du\, uv\, \mathcal{T}(u,v)$$
$$\times \left[e^{-\frac{(vk-k_*)^2}{2\sigma^2}} + 2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{vk}\sqrt{\frac{\pi}{2}}\mathrm{erf}\left(\frac{vk}{2\sigma}\right)\right]$$
$$\times \left[e^{-\frac{(uk-k_*)^2}{2\sigma^2}} + \boxed{2\mathcal{A}_{\mathcal{R}} F_{\rm NL}^2 \frac{\sigma}{uk}\sqrt{\frac{\pi}{2}}\mathrm{erf}\left(\frac{uk}{2\sigma}\right)}\right]. \quad (7)$$

where the integral kernel

$$\mathcal{T}(u,v) = \frac{1}{4}\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2\left(\frac{u^2+v^2-3}{2uv}\right)^2$$
$$\times \left\{\left(-2+\frac{u^2+v^2-3}{2uv}\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2\right.$$
$$\left. + \pi^2\left(\frac{u^2+v^2-3}{2uv}\right)^2\Theta\left(u+v-\sqrt{3}\right)\right\}. \quad (8)$$

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The Gaussian case

If the non-Gaussian contribution is small, the leading order is is given by the Gaussian integral. For a delta function like peak, the main contribution comes from the neighborhood of

$u \sim v \sim k_*/k,$

$$\Omega_{\rm GW}^{(0)} \simeq 6\mathcal{A}_{\mathcal{R}}^2 \left(\frac{k}{k_*}\right)^2 \mathcal{T}\left(\frac{k_*}{k}, \frac{k_*}{k}\right) \Theta(2k_* - k).$$

when $k \ll k_*$:

$\Omega_{\rm GW}^{(0)} \propto k^2$, with a peak about $\Omega_{\rm GW,peak}^{(0)} \simeq 21.0\mathcal{A}_{\mathcal{R}}^2$ at $k_p^{(0)} \sim (2/\sqrt{3})k_*$.

The non-Gaussian case

When $\mathcal{A}_\mathcal{R} F_{\rm NL}^2 \gtrsim \mathcal{O}(1)$, the contribution from the non-Gaussian part dominates the tensor power spectrum:

$$\begin{aligned}
\Omega_{\rm GW}^{(2)} =& 6\mathcal{A_R}^3 F_{\rm NL}^2 \left(\frac{k}{k_*}\right)^3 \Theta(3k_* - k) \\
&\times \left[\int_{|1-k_*/k|}^{\min(1+k_*/k, 2k_*/k)} du\, \mathcal{T}\left(u, \frac{k_*}{k}\right)\right. \\
&\left. + \int_{\max(0,|k_*/k-1|)}^{\min(2k_*/k, 1+k_*/k)} dv\, \mathcal{T}\left(\frac{k_*}{k}, v\right)\right]. \\
\Omega_{\rm GW}^{(4)} =& 6\mathcal{A_R}^4 F_{\rm NL}^4 \left(\frac{k}{k_*}\right)^4 \int_0^\infty dv \int_{|1-v|}^{1+v} du \\
&\times \mathcal{T}(u,v)\Theta\left(2k_* - vk\right)\Theta\left(2k_* - uk\right).
\end{aligned}$$

If $\mathcal{A}_\mathcal{R} F_{\rm NL}^2 \gg 1$, the second part dominates.

The scaling law when k<<k_*:

$$\Omega_{\rm GW}^{(4)} \simeq 89.6\,(\mathcal{A}_{\mathcal{R}}F_{\rm NL})^4\left(\frac{k}{k_*}\right)^3\left[1+\cdots\right]\Theta(4k_*-k).$$

The GWs induced by non-Gaussian scalar perturbations are easily distinguishable if they dominates:

$$\frac{\Omega_{\rm GW}^{(4)}}{\Omega_{\rm GW}^{(0)}} \sim 4.3\mathcal{A}_{\mathcal{R}}^2F_{\rm NL}^4.$$

Clearly if this is larger than one, we can see the effect of the non-Gaussianity. If $F_{\rm NL} \gtrsim 10$ will be enough for a peak amplitude of $\mathcal{A}_{\mathcal{R}} \sim 10^{-2}$.

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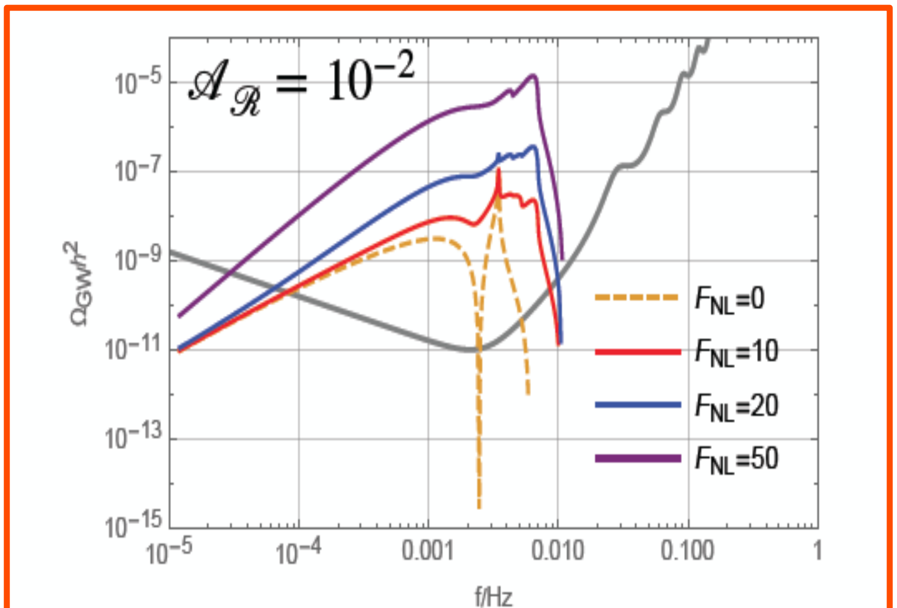
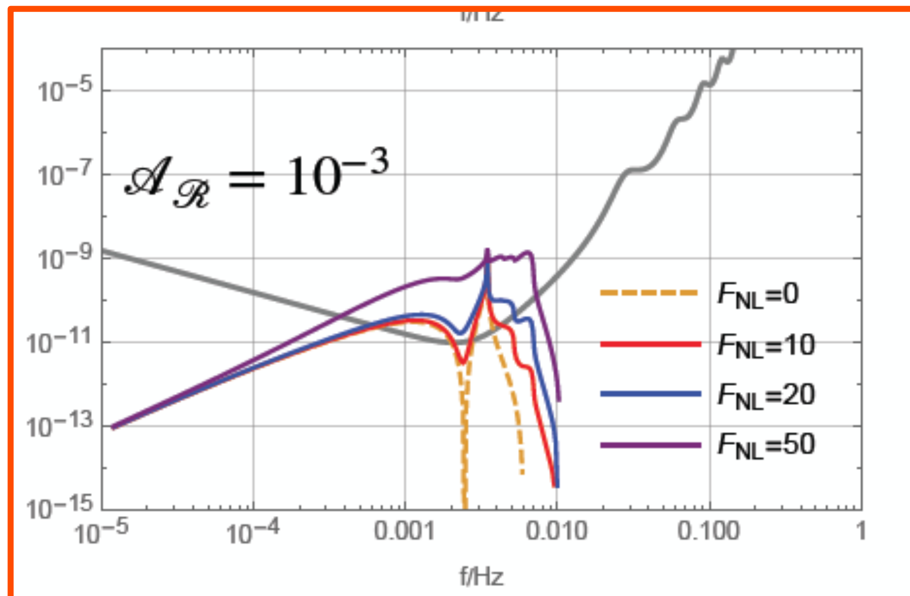
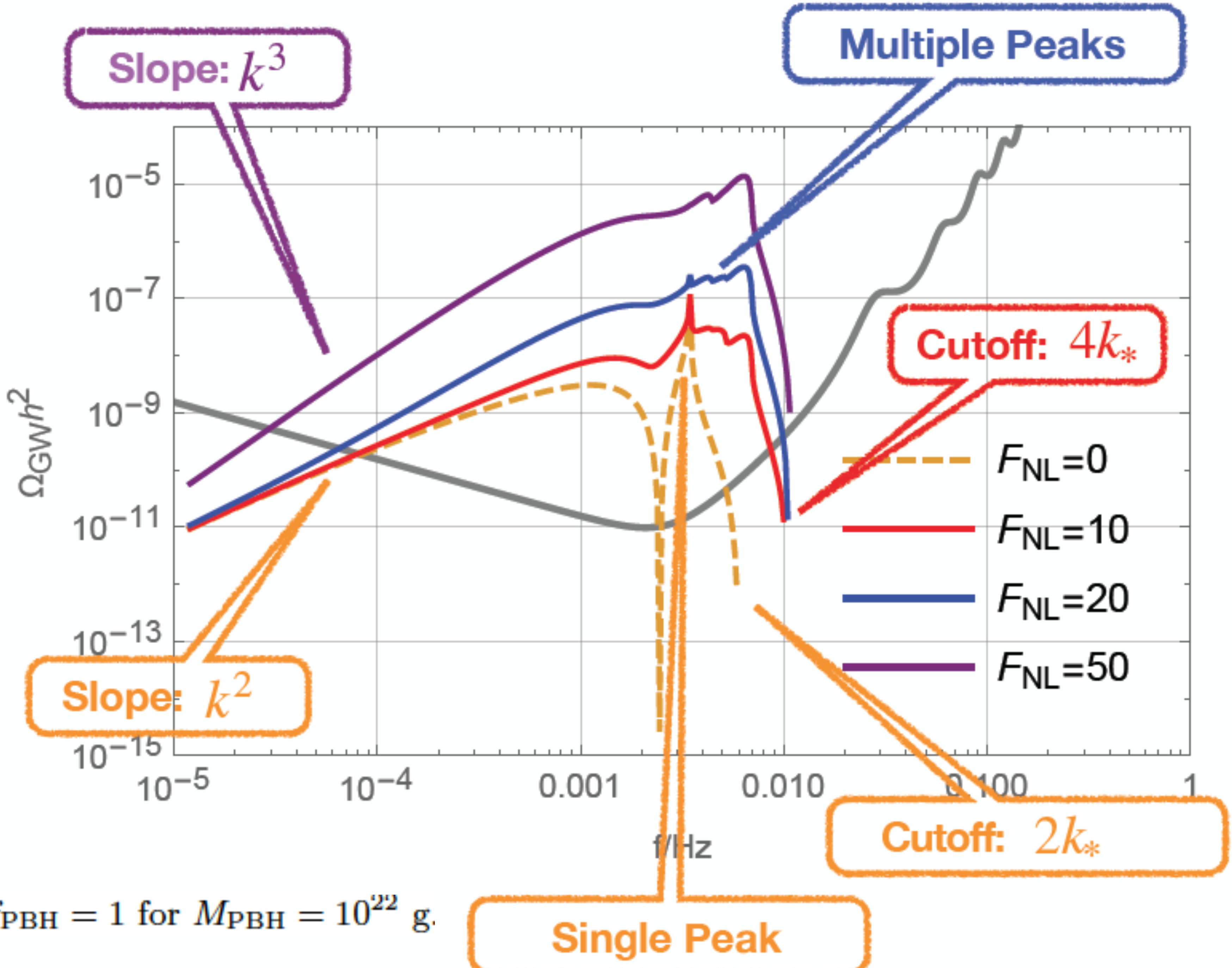
As a result, if $\mathcal{A}_{\mathcal{R}}^2 F_{\rm NL}^4 \gtrsim 1$, then $\Omega_{\rm GW}^{(4)}$ and $\Omega_{\rm GW}^{(2)}$ are larger than $\Omega_{\rm GW}^{(0)}$, we find a series of peaks from the resonances around $k_p^{(0)} \sim (2/\sqrt{3})k_*$, $k_p^{(2)} \sim \sqrt{3}k_*$, and $k_p^{(4)} \sim (4/\sqrt{3})k_*$, which can be recognized as a smoking gun of the primordial non-Gaussianity at scale k_*.



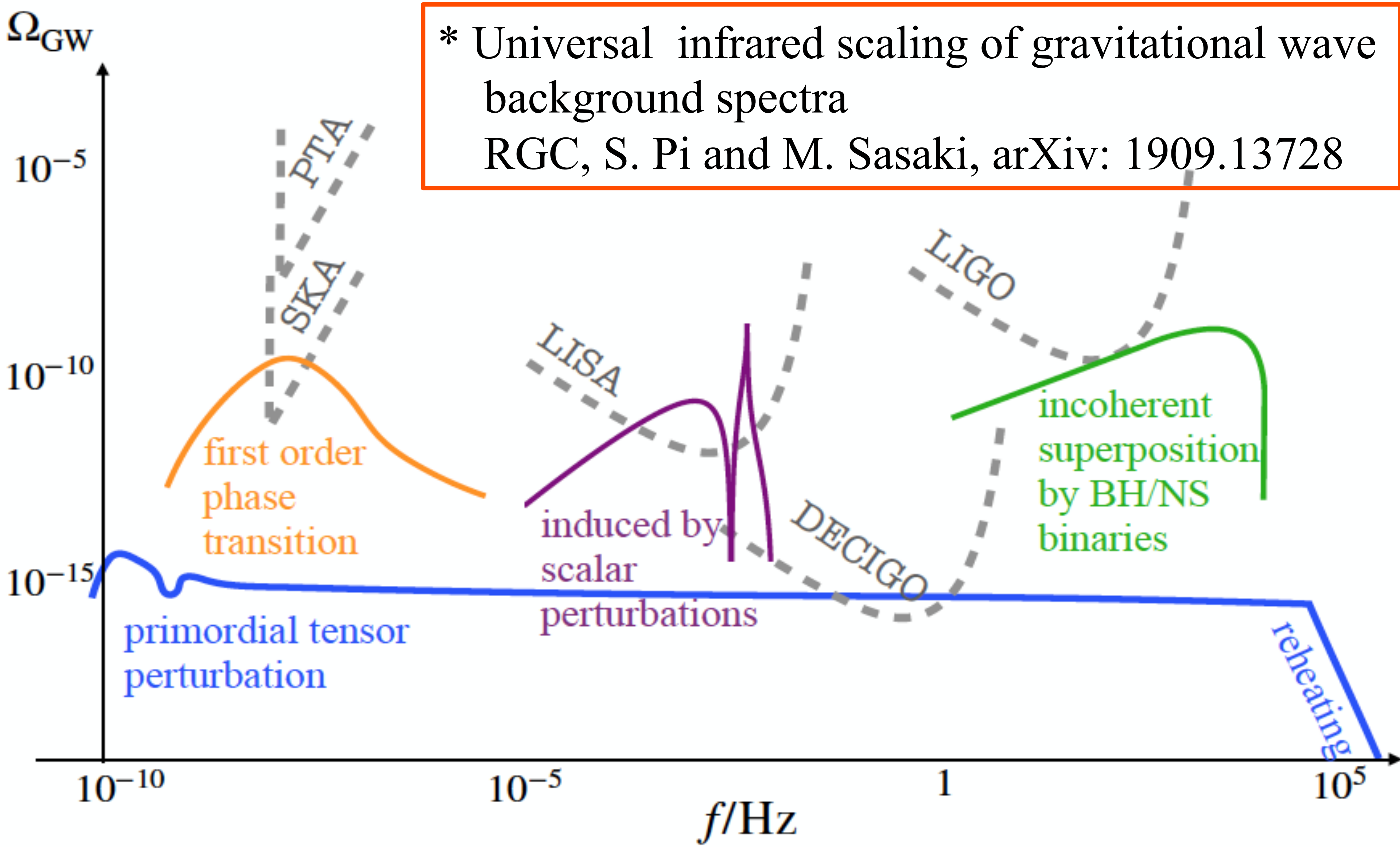


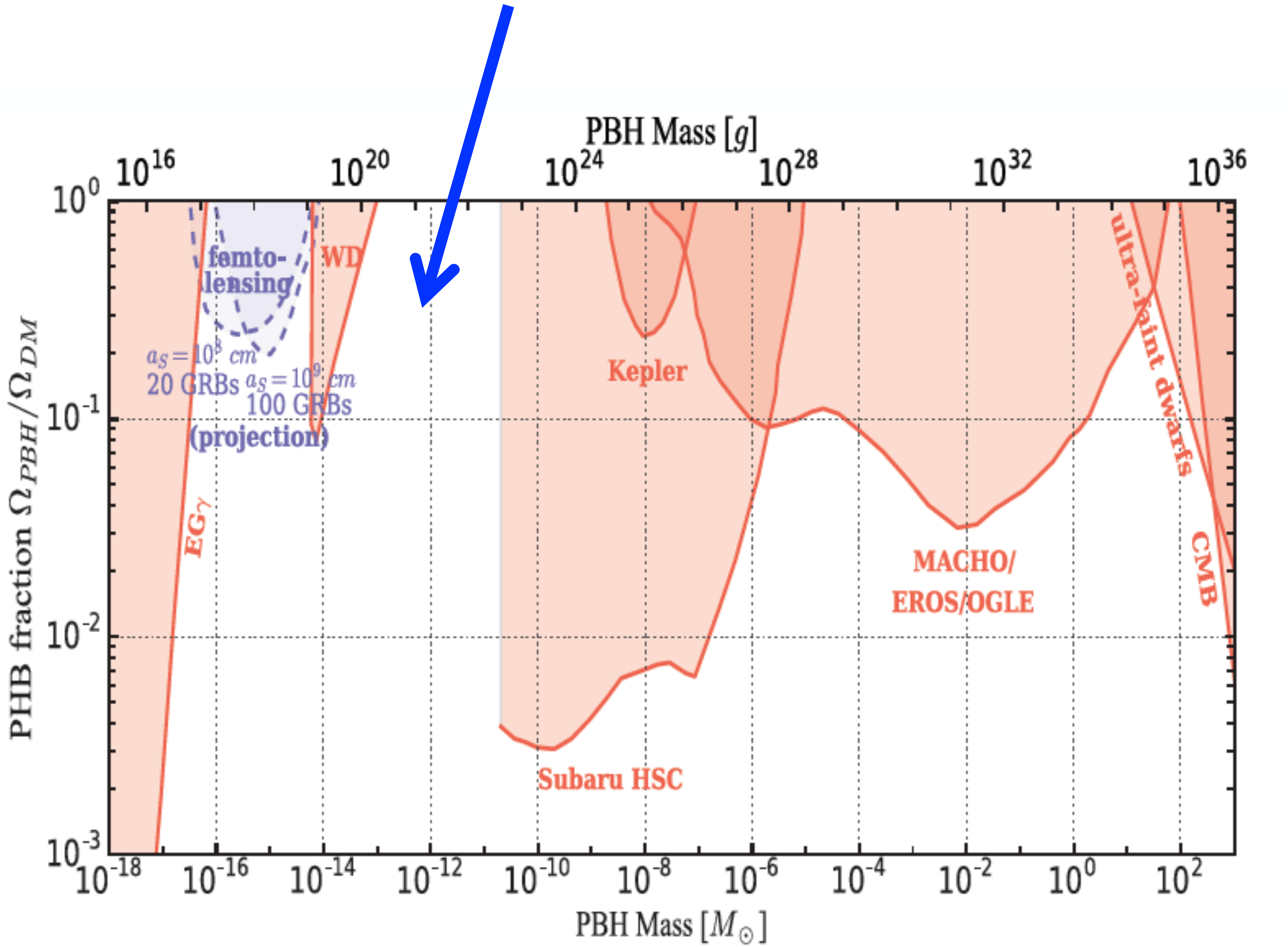


$f_{\rm PBH} = 1$ for $M_{\rm PBH} = 10^{22}$ g.

The stochastic GWs background:

* Universal infrared scaling of gravitational wave background spectra
  RGC, S. Pi and M. Sasaki, arXiv: 1909.13728

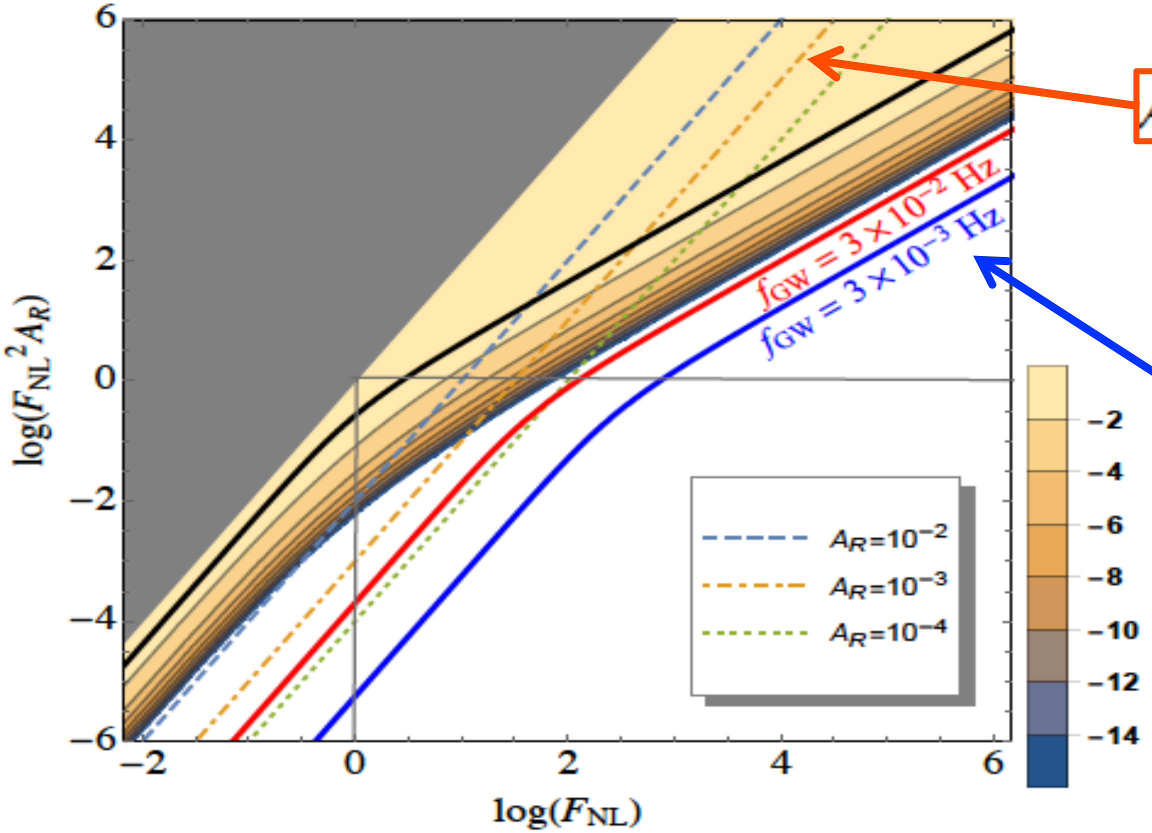




PBH Mass [g]
10^{16}
10^{20}
10^{24}
10^{28}
10^{32}
10^{36}
10^{0}
10^{-1}
10^{-2}
10^{-3}
PHB fraction Ω_{PBH}/Ω_{DM}
femto-
lensing
WD
$a_S = 10^8$ cm
20 GRBs
$a_S = 10^9$ cm
100 GRBs
(projection)
EG_γ
Kepler
ultra-faint dwarfs
MACHO/
EROS/OGLE
CMB
Subaru HSC
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
PBH Mass $[M_\odot]$

PBH mass fraction:

$$\beta \lesssim 8.6 \times 10^{-15} \left(\frac{3 \times 10^{-3}\ \mathrm{Hz}}{f_{\mathrm{GW}}} \right). \qquad f_{\mathrm{GW}} \sim 3\ \mathrm{Hz} \left(\frac{M_{\mathrm{PBH}}}{10^{16}\mathrm{g}} \right)^{-1/2},$$

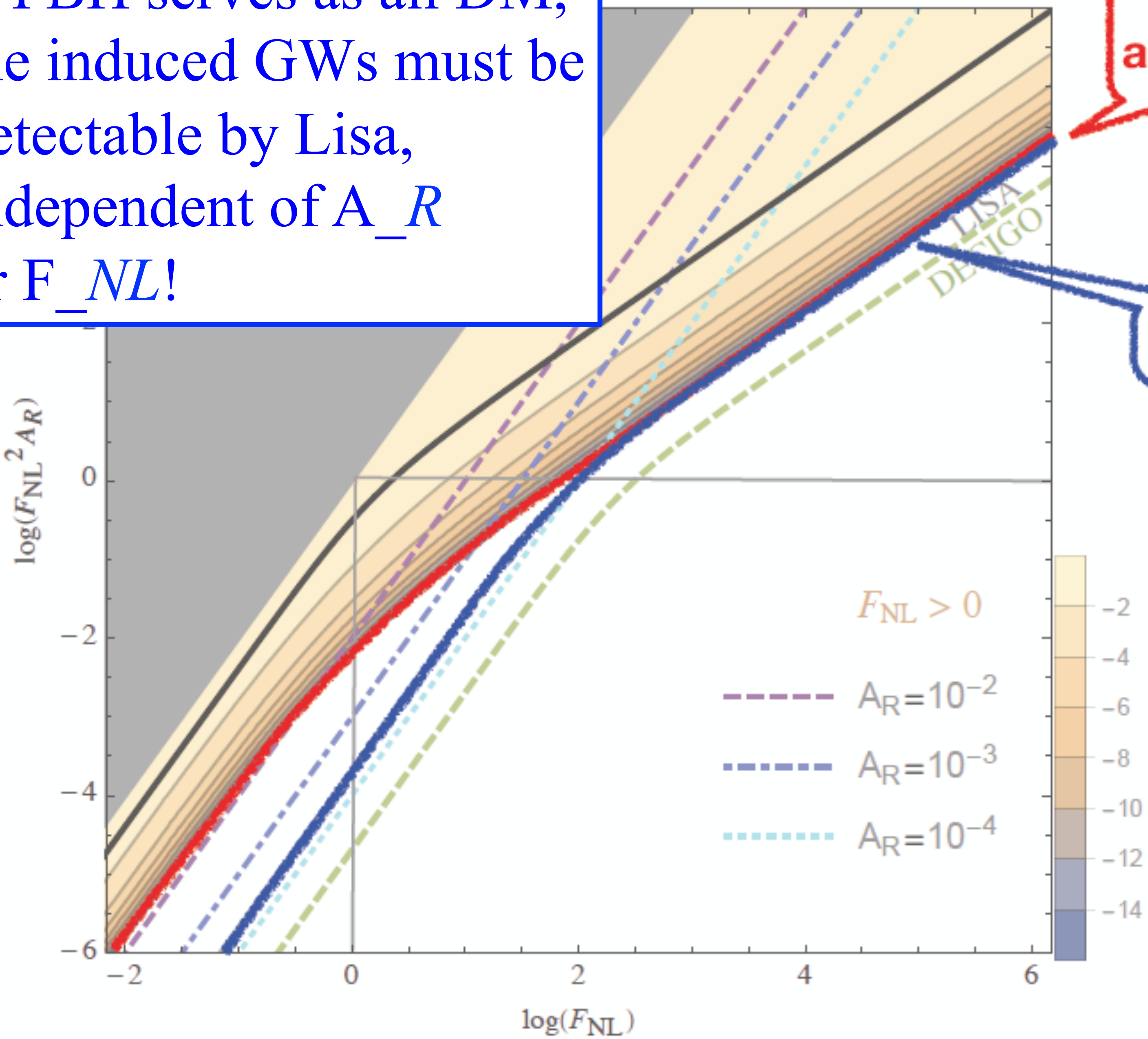


$A_{\mathcal{R}} = 10^{-2},\ 10^{-3},$ and 10^{-4}

$M_{\mathrm{PBH}} = \{10^{20}\ \mathrm{g}, 10^{22}\ \mathrm{g}\}$

The sensitivity bound of Lisa

If PBH serves as all DM, the induced GWs must be detectable by Lisa, independent of A_*R* or F_*NL*!



PBH as all Dark Matter

LISA bound

Conclusions

- Induced GW is a very important source of SGWB.
- GWs induced by non-Gaussian scalar perturbations: k^3-slope, multiple peaks, cutoff.
- If PBHs can serve as all the DM, induced GWs must be detectable by LISA, no matter how small $\mathscr{A}_{\mathscr{R}}$ or $f_{\rm NL}$ is.

3) Resonant multiple peaks in the induced GWs

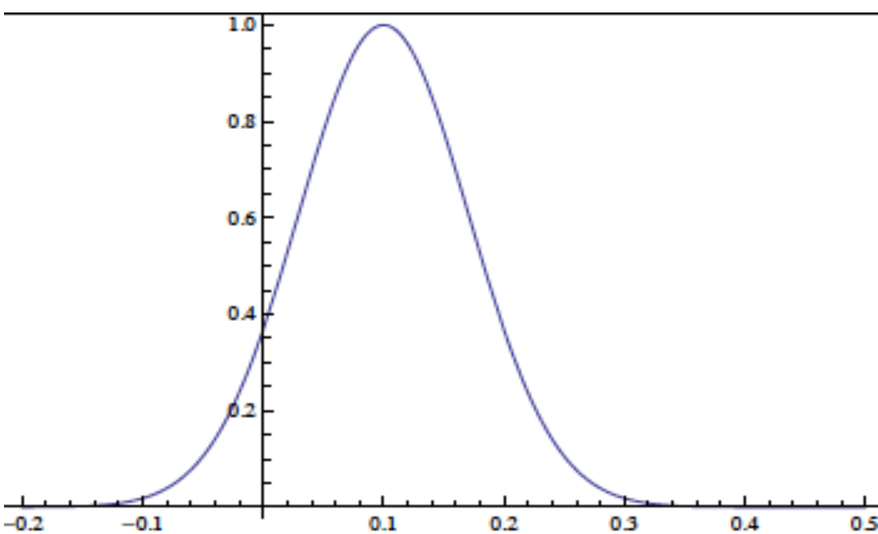
RGC, S. Pi, S.J. Wang and X.Y. Yang, arXiv. 1901.10152
(JCAP 05 (2019) 013)

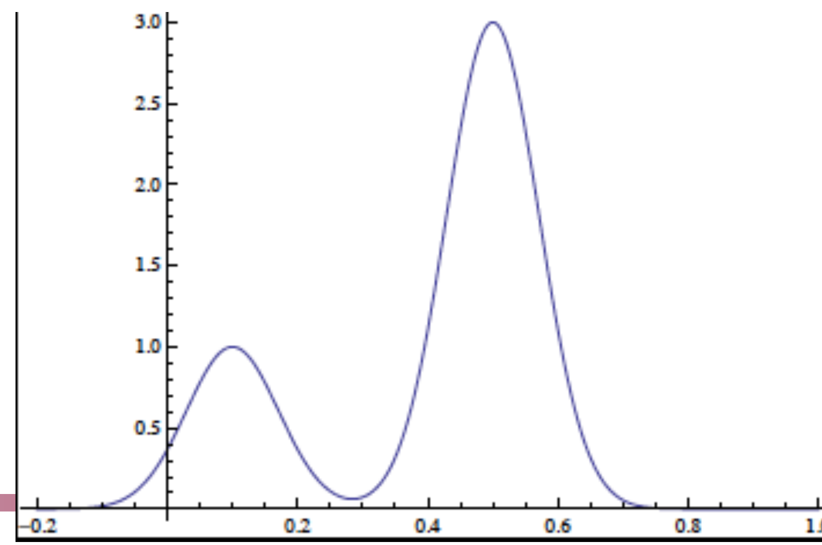
If Curvature perturbation:

Something New?

$$P_\phi(k) = \frac{A_\phi}{(2\pi)^{3/2} 2\sigma k_*^2} e^{-\frac{(k-k_*)^2}{2\sigma^2}} .$$

$$\tilde{P}_\phi(k) = \sum_{i=1}^{n} \frac{A_{\phi i} k_{*i}}{\sqrt{2\pi}\sigma} e^{-\frac{(k-k_{*i})^2}{2\sigma^2}} ,$$





Gaussian & non-Gaussian perturbations

1） The case with multiple delta peaks

$$\tilde{P}_\phi(k) = \sum_{i=1}^{n} A_{\phi i}\delta\left(\ln\frac{k}{k_{*i}}\right).$$

2） The case with multiple sigma peaks

$$\tilde{P}_\phi(k) = \sum_{i=1}^{n} \frac{A_{\phi i}k_{*i}}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(k-k_{*i})^2}{2\sigma^2}},$$

The energy density spectrum:

$$\delta\text{-peak}: \quad \tilde{P}_\phi(k) = \sum_{i=1}^{n} A_{\phi i}\delta(\ln\frac{k}{k_{*i}});$$

$$\Omega_{\mathrm{GW}}^{n,\delta}(k) = \frac{1}{24}\sum_{i,j=1}^{n} A_{\phi i}A_{\phi j}\frac{k_{*i}k_{*j}}{k^2}\mathcal{T}\left(\frac{k_{*i}}{k},\frac{k_{*j}}{k}\right)\mathcal{F}^{\delta}(k;k_{*i},k_{*j});$$

$$\sigma\text{-peak}: \quad \tilde{P}_\phi(k) = \sum_{i=1}^{n}\frac{A_{\phi i}k_{*i}}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(k-k_{*i})^2}{2\sigma^2}};$$

$$\Omega_{\mathrm{GW}}^{n,\sigma}(k) = \frac{1}{24}\sum_{i,j=1}^{n} A_{\phi i}A_{\phi j}\frac{k_{*i}k_{*j}}{k^2}\mathcal{T}\left(\frac{k_{*i}}{k},\frac{k_{*j}}{k}\right)\mathcal{F}^{\sigma}(k;k_{*i},k_{*j}),$$

Abstract. We identify analytically a multiple-peak structure in the energy-density spectrum of induced gravitational waves (GWs) generated at second-order from a primordial scalar perturbations also with multiple (n) peaks at small scales k_{*i}. The energy-density spectrum of induced GWs exhibits at most C_{n+1}^2 and at least n peaks at wave-vectors $k_{ij} \equiv (k_{*i}+k_{*j})/\sqrt{3}$ due to resonant amplification, and, under the narrow-width approximation, it contains an universal factor that can be interpreted as a result of momentum conservation. We also extend these discussions to the case of non-Gaussian perturbations.

The case with triple peaks

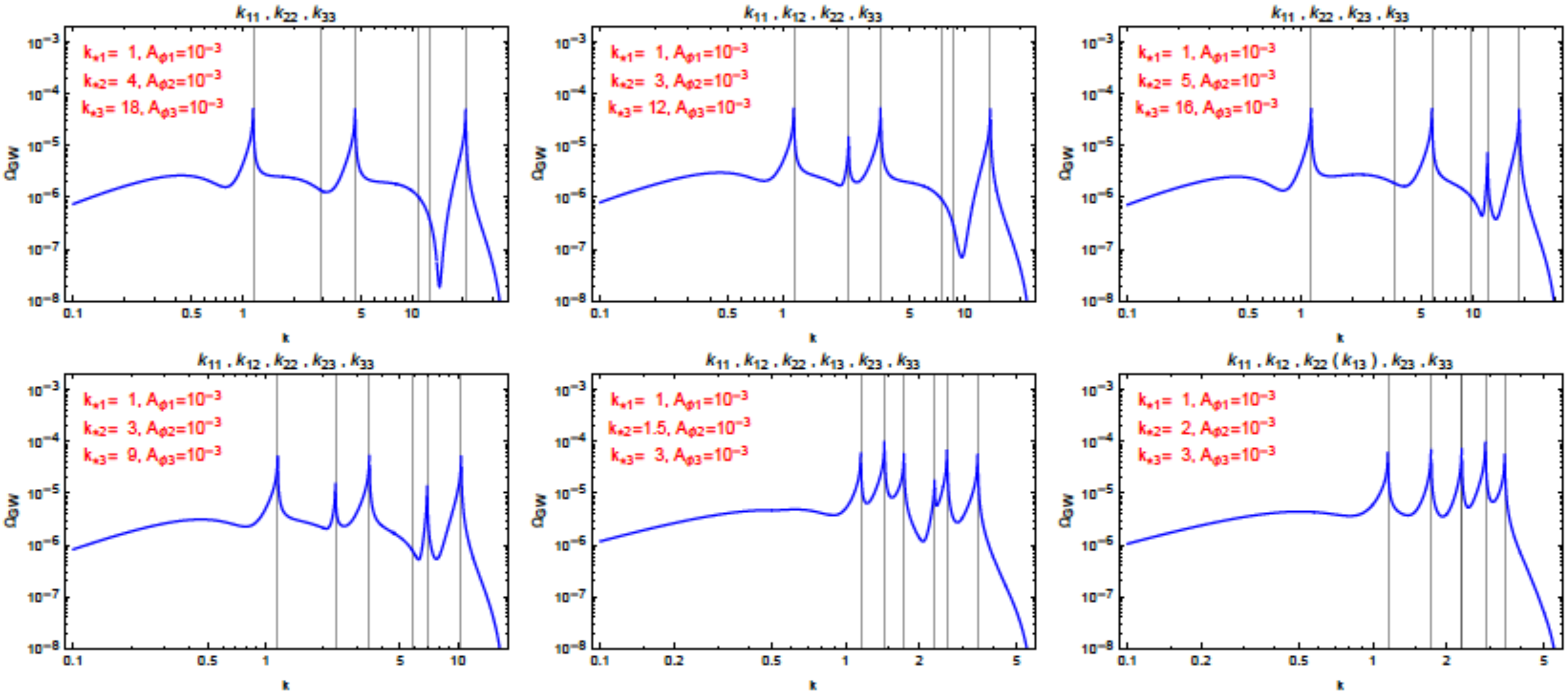


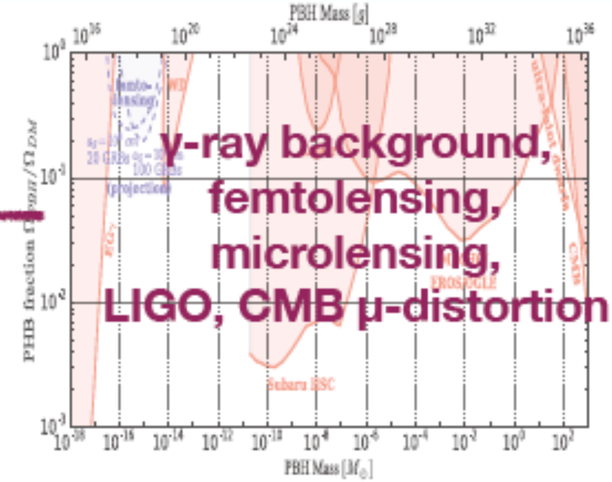
Figure 3. The energy-density spectrum of induced GWs from scalar perturbations with triple δ-peaks at $k_{*1} < k_{*2} < k_{*3}$. The gray lines denote the positions of those would-be peaks at k_{ij} with $i, j = 1, 2, 3$.

Induced GWs

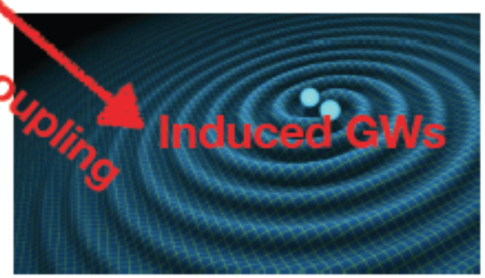
Peak of scalar perturbation on small scales

Peak Theory

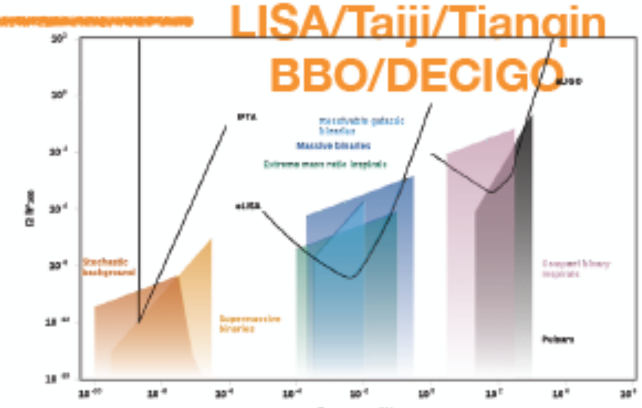
?



secondary coupling



LIGO/VIRGO/KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO



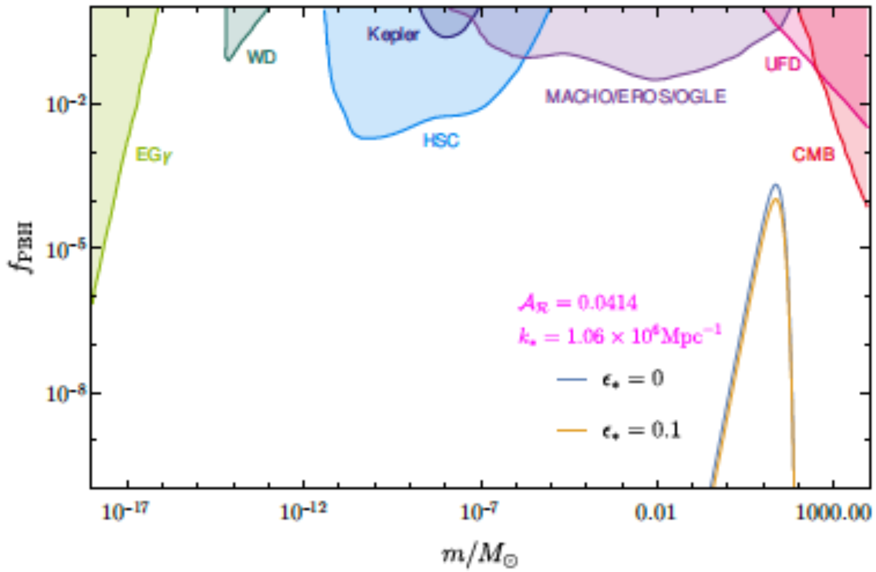
Saito & Yokoyama, 0812.4339

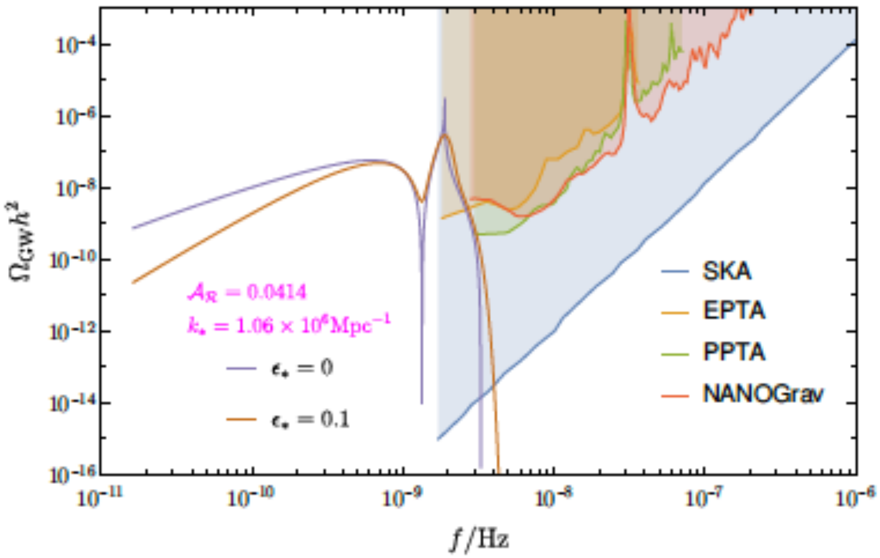
4) PTA Constraints on the induced gravitational waves

We consider the case where the power spectrum of curvature Perturbations has a narrow Gaussian peak at some scale k_* with width sigma_* << k_*:

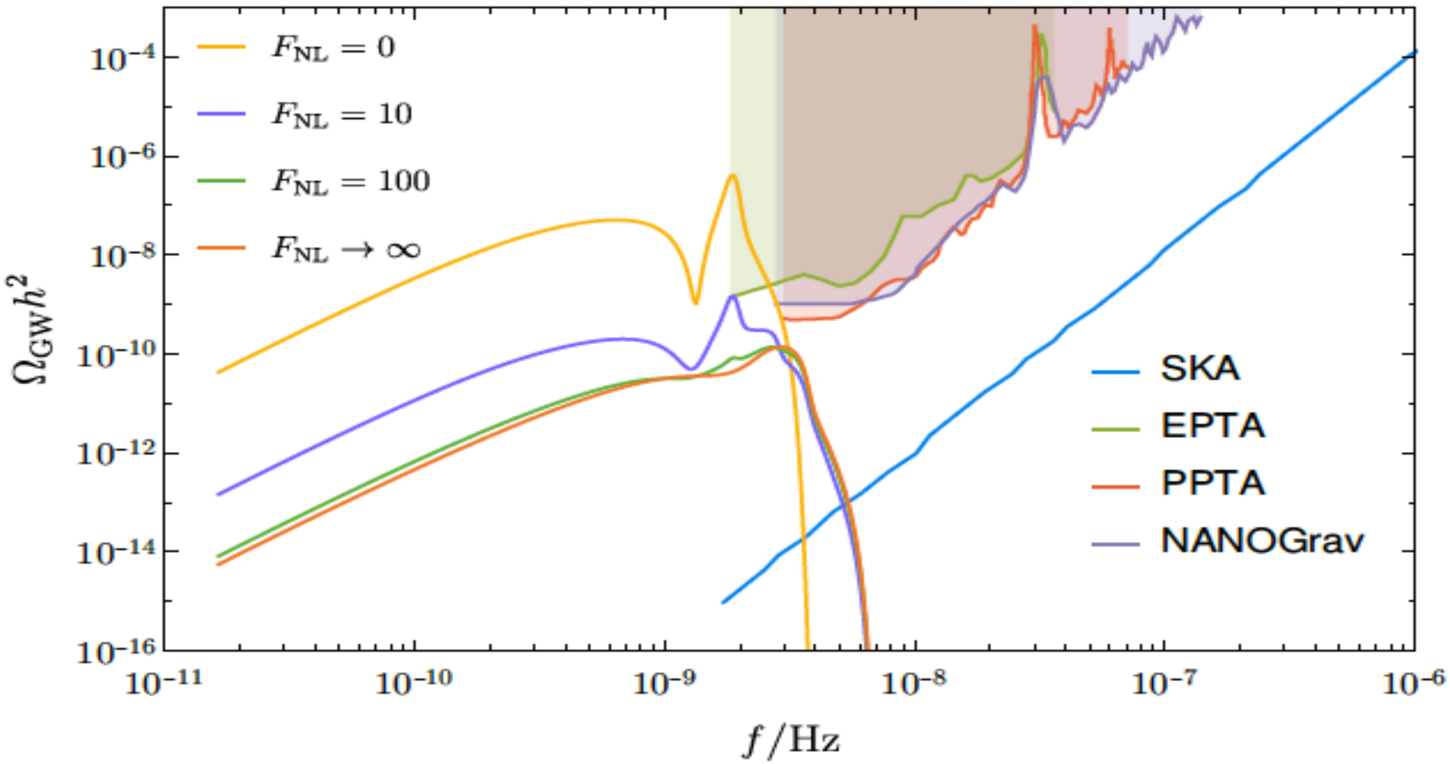
$$\mathcal{P}_{\mathcal{R}}(k) = \frac{\mathcal{A}_{\mathcal{R}} k_*}{\sqrt{2\pi}\sigma_*} \exp\left[-\frac{(k-k_*)^2}{2\sigma_*^2}\right].$$

rule out?





Non-Gaussian effect?

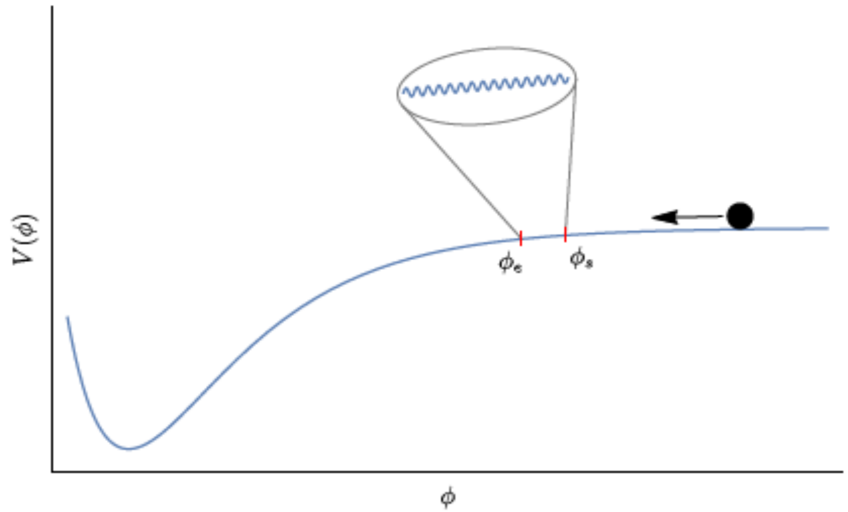


R.G. Cai, S. Pi, S.J. Wang and X.Y. Yang:
arXiv: 1907.06372, JCAP 1910 (2019) 059

5) Primordial black holes and gravitational waves from parametric amplification of curvature perturbations RG Cai, et al, arXiv: 1912.10437

Model:

$$
\begin{aligned}
V(\phi) &= \bar{V}(\phi) + \delta V(\phi) \\
&= \bar{V}(\phi) + \xi \cos(\phi/\phi_*)\Theta(\phi; \phi_s, \phi_e),
\end{aligned}
$$



The Mukhanov-Sasaki equation:

$$\ddot{u}_k + H\dot{u}_k + \left[\frac{k^2}{a^2} - \frac{\xi}{\phi_*^2}\cos(\frac{\phi}{\phi_*})\right]u_k = 0.$$

For the small scales (k>> a H)

$$\frac{d^2u_k}{dx^2} + [A_k(x) - 2q\cos 2x]\,u_k = 0,$$

where $k_* = \frac{\dot{\phi}_s}{2\phi_*}$, $x = k_* t + \frac{\phi_s - \dot{\phi}_s t_s}{2\phi_*}$, $A_k(x) = \frac{k^2}{k_*^2 a^2}$, $q = 2\xi/\dot{\phi}_s^2$.

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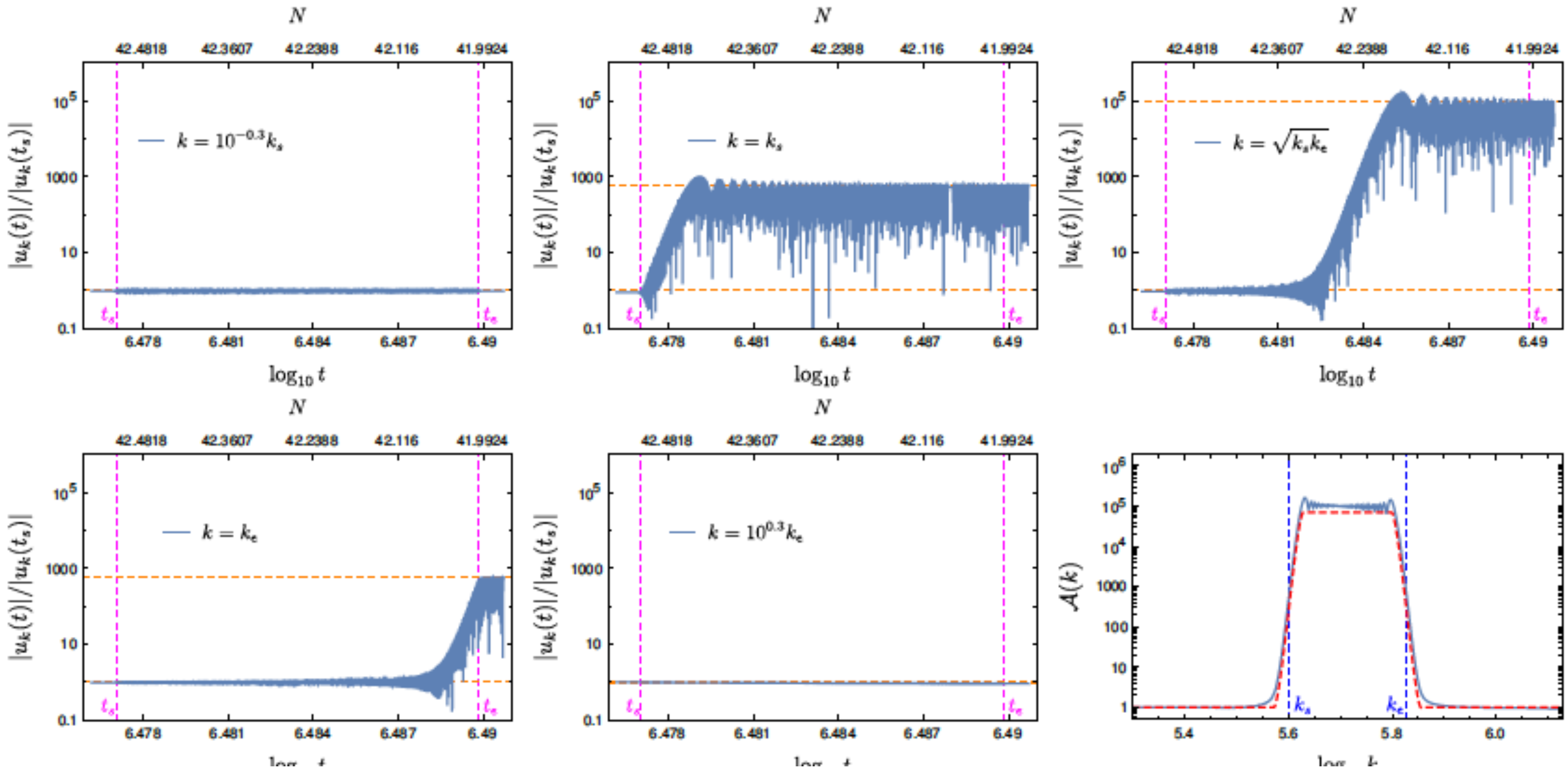
Some examples:



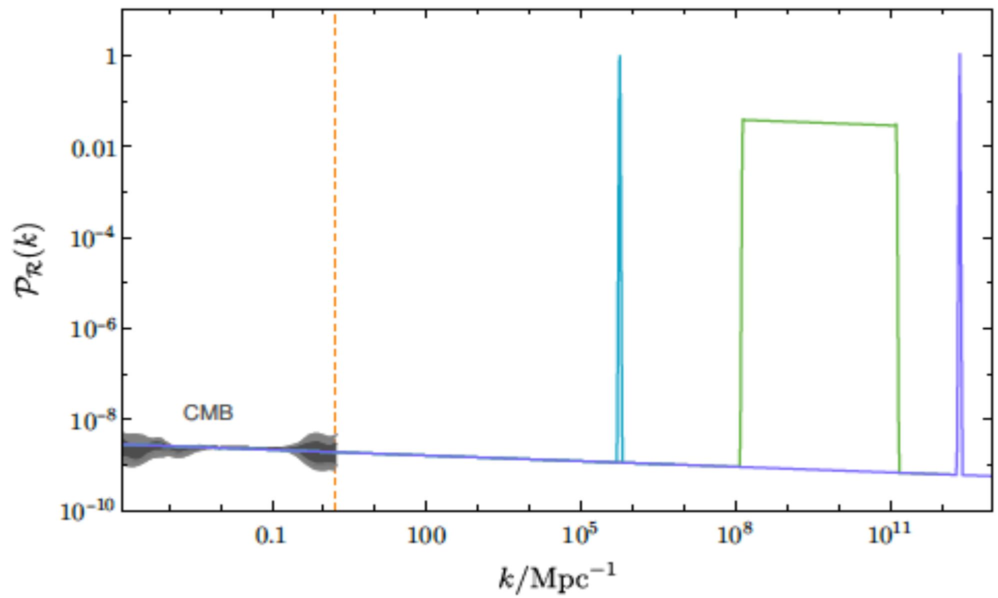
An example: the Starobinsky potential

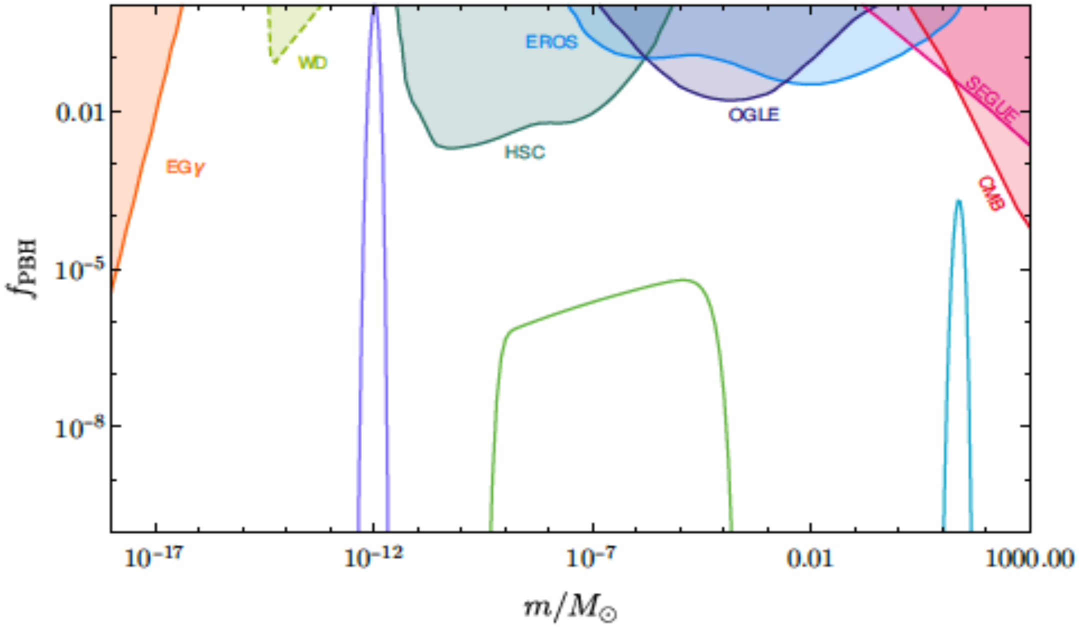
$$\bar{V}(\phi) = \Lambda^4 \left[1 - \exp(-\sqrt{2/3}\phi)\right]^2, \qquad (10)$$

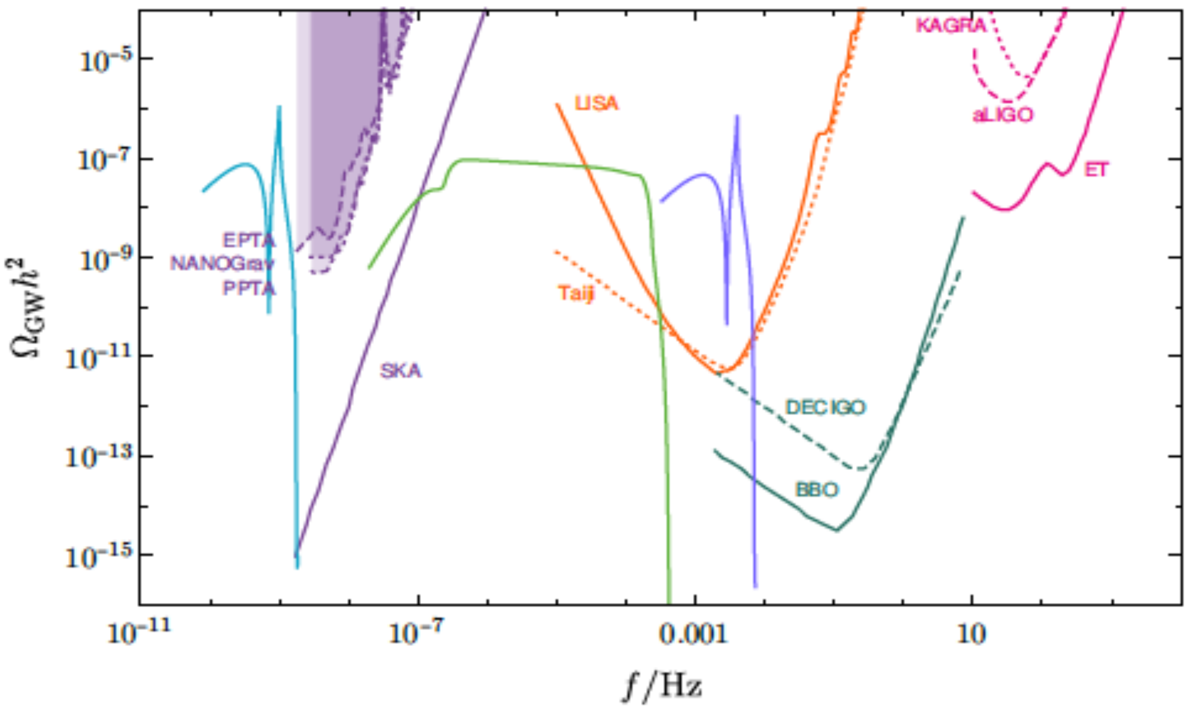
where $\Lambda = 0.0032$ is set to match the CMB experiments and we set the e-folding number when the pivot scale leaves horizon as $N_{\rm CMB} = 60$ [47].

Set	ξ	ϕ_*	ϕ_s	ϕ_e
1	1.7×10^{-15}	8×10^{-6}	4.9878	4.9731
2	1.59×10^{-15}	7.95×10^{-6}	4.9872	4.9837
3	2.08×10^{-15}	9.37×10^{-6}	4.8219	4.5725
4	3.9×10^{-15}	1.24×10^{-5}	4.461	4.456

TABLE I. The parameter sets we choose as examples.







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Thanks!