Identification of Lensed Gravitational Waves with Deep Learning

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In collaboration with

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Motivation

- Expect "gravitational lensing" effect in gravitational waves (GWs) like gravitational lensing in astronomical EM observations.
- If time delay were small enough to make "beating pattern", may identify lensed GW from a spectrogram, time-frequency map, of the chirp signal of GWs from compact binary mergers by using Machine Learning or Deep Learning.

Motivation

- Expect "gravitational lensing" effect in gravitational waves (GWs) like gravitational lensing in astronomical EM observations.
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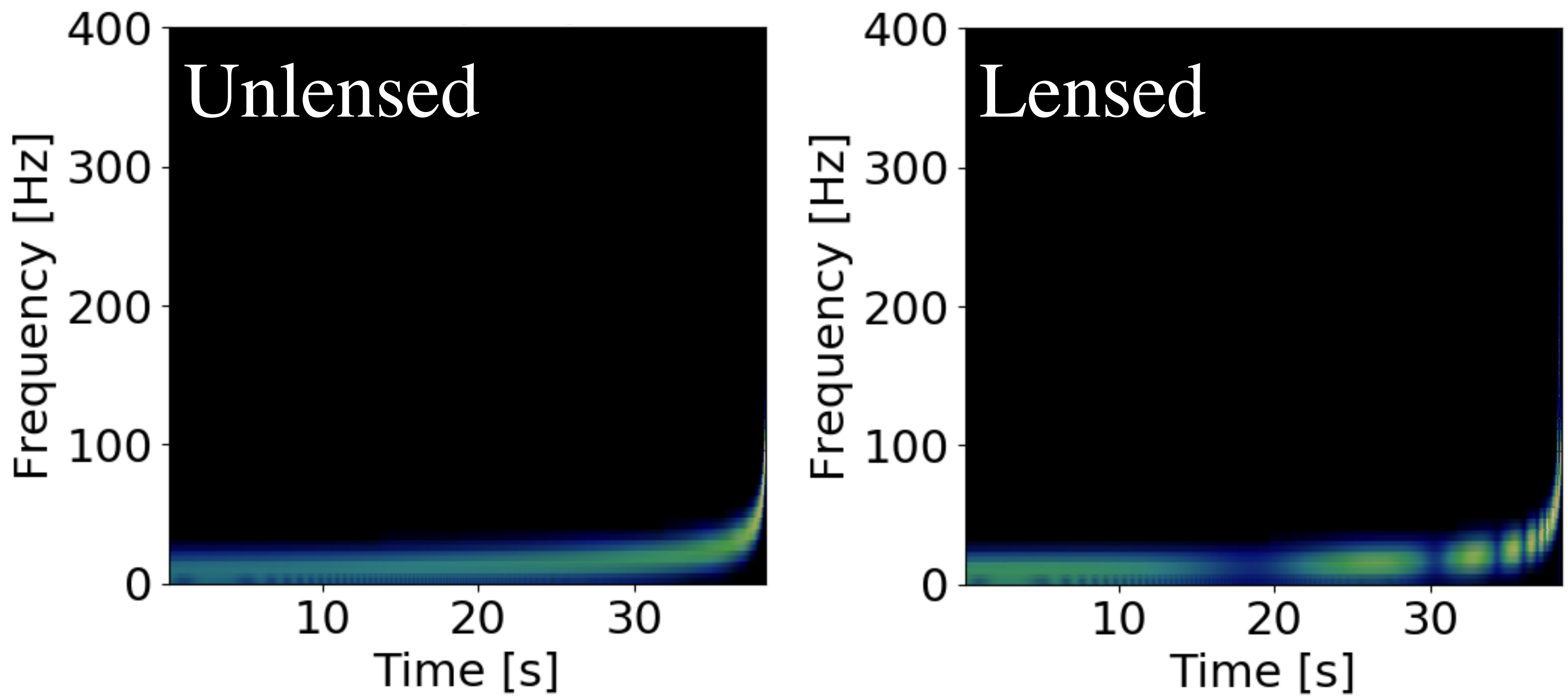
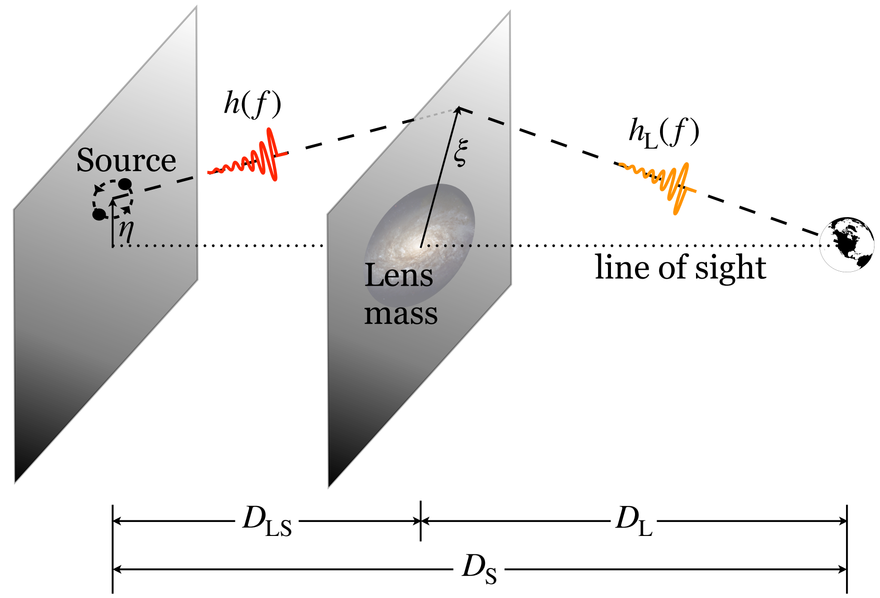


Lensing Model [Takahashi & Nakamura, ApJ 2003]



- Strain amplitude of lensed GW in frequency domain

$$h_{\rm lens}(f) = F(f)h(f)$$

where $F(f)$ is the ***amplification factor*** which is written (under the geometrical optics limit) as

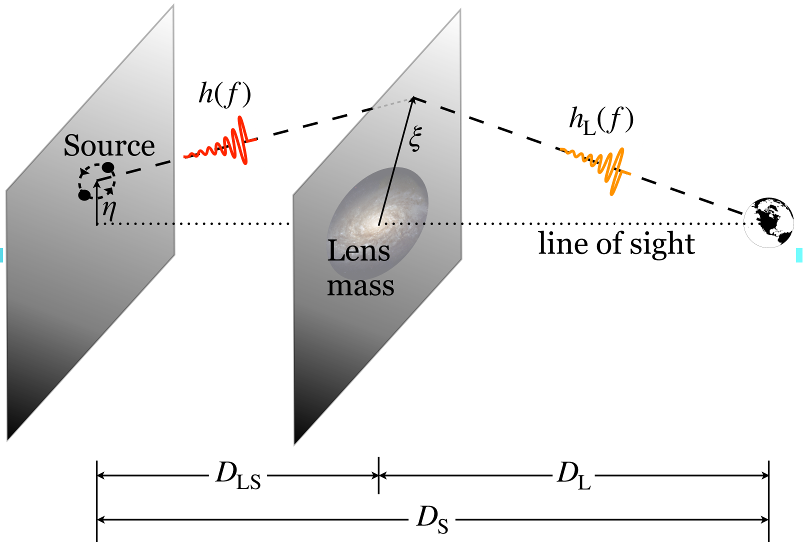
$$F(f) = \sum_j |\mu_j|^{1/2} e^{2\pi i f \Delta t_{d,j} - i\pi n_j}$$

model dependent number

time delay of j-th image

magnification factor

Lensing Model [Takahashi & Nakamura, ApJ 2003]



- Point-Mass (PM) Lens
  - Lens mass: 2-dim Dirac delta function on lens plane

$$F(f) = |\mu_+|^{1/2} - i\,|\mu_-|^{1/2} e^{2\pi i f \Delta t_d}$$

$$M_{Lz} = M_L(1+z)$$

$$\mu_\pm = \frac{1}{2} \pm \frac{(y^2+2)}{(2y\beta)} \qquad \Delta t_d = 4M_{Lz}\left[\frac{y\beta}{2} + \ln\left(\frac{\beta+y}{\beta-y}\right)\right]$$

$$\beta = \sqrt{y^2+4}$$

$$y = \frac{\eta D_L}{\xi_0 D_S}$$

- Singular Isothermal Sphere (SIS)
  - Lens mass: circular and symmetric distribution on lens plane

$$F(f) = \begin{cases} |\mu_+|^{1/2} - i|\mu_-|^{1/2} e^{2\pi i f \Delta t_d} & \text{if } y < 1, \\ |\mu_+|^{1/2} & \text{if } y \geq 1 \end{cases}$$

$$\mu_\pm = \frac{1}{y} \pm 1 \qquad \Delta t_d = 8M_{Lz}y \qquad y = \frac{\eta D_L}{\xi_0 D_S}$$

Strategy

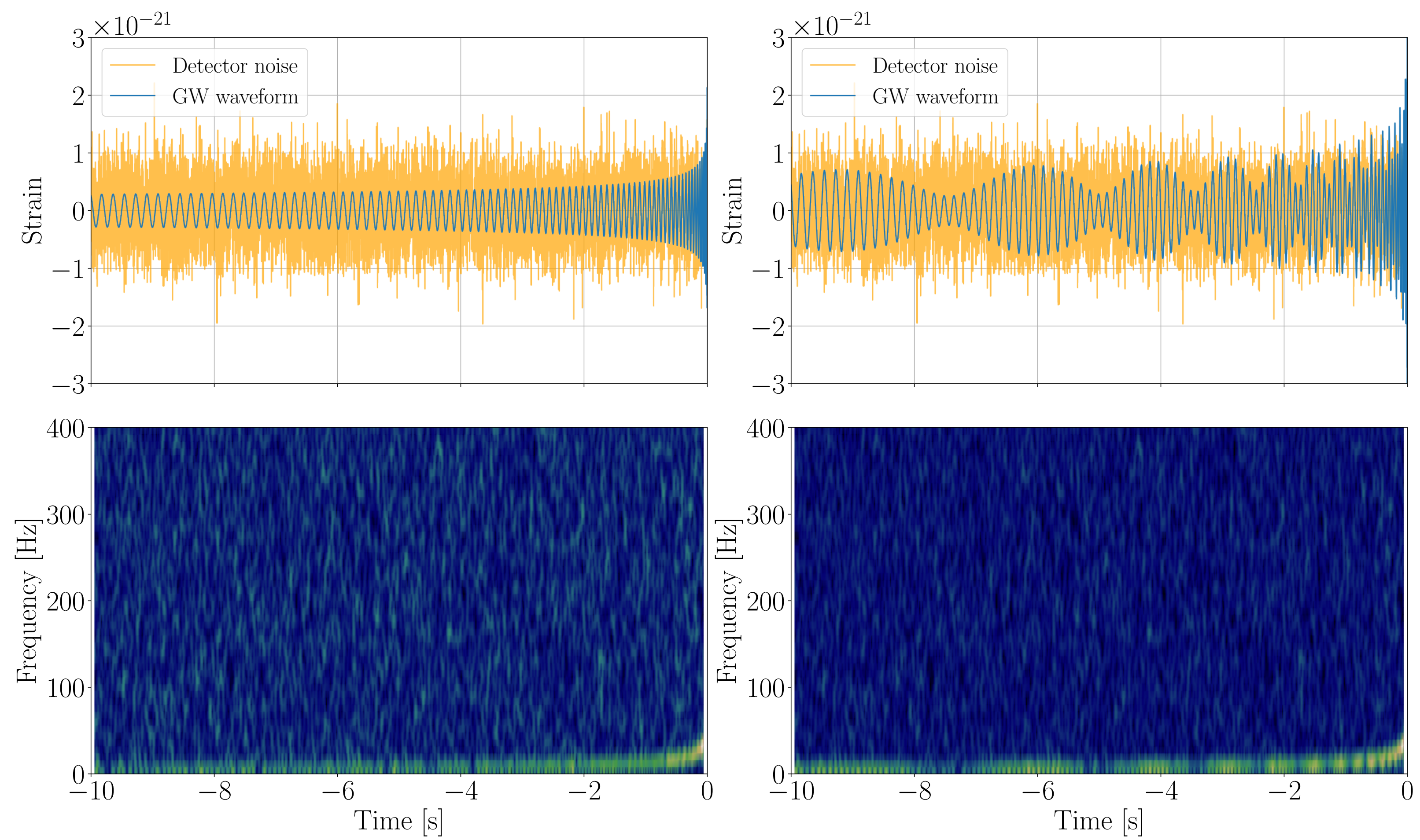
- **Classification**
  - classifies lensed signal from unlensed signal.
  - w/ Scikit-Learn and TensorFlow

↓

- **Regression**
  - estimates lensing parameters
    - source mass, lens mass, distance, and so on.
  - w/ TensorFlow

Classification with Machine Learning w/ Scikit-Learn [Singh et al., AJUR 2019, arXiv:1810.07888]

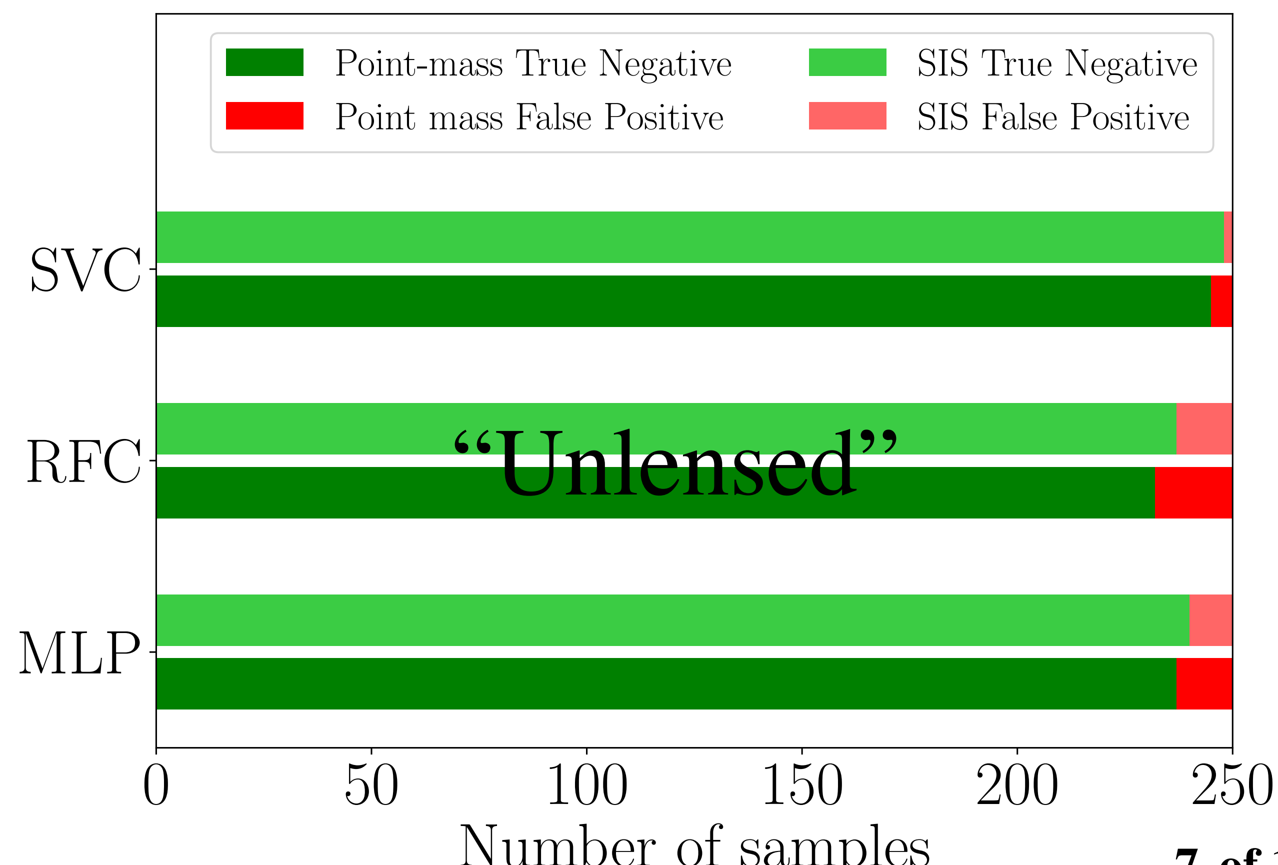
- Input data: 2-dim spectrogram images using Short-Time-Fourier-Transform
  - 2 classes: unlensed vs. lensed
  - # of samples: 1000 samples for unlensed / 1000 samples for lensed (PM or SIS)
  - Parameters
    - m_1, m_2: 4-35 solar mass
    - D_L: 10-1000 Mpc
    - D_{LS}: 10-1000 Mpc
    - M_L: 10-10^7 solar mass
    - ε: 0-0.5 pc
    - redshift factor, z: 0-2
  - Noise: random white
    - SNR: ≤ 80 (c.f. ≤ 23.6 for O1&O2 BBHs in GWTC-1)



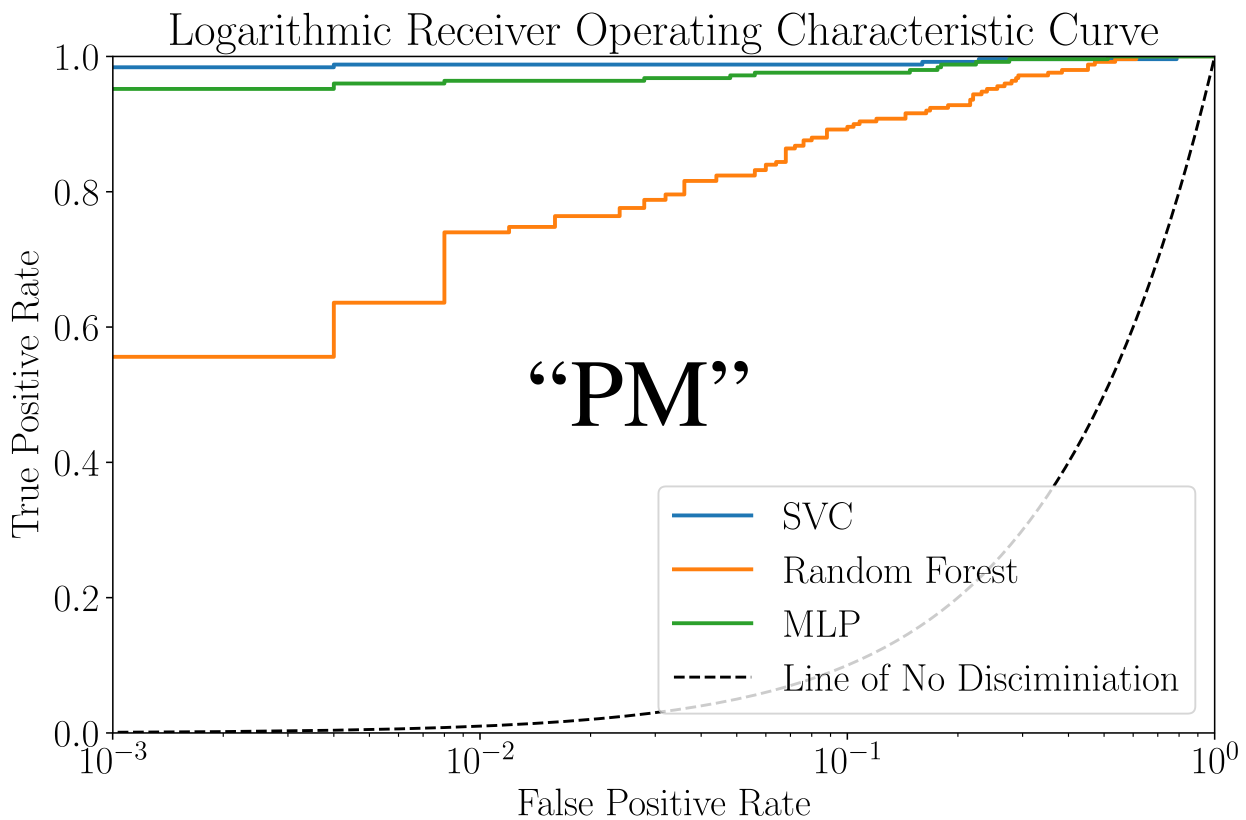
Classification Performance [Singh et al., AJUR 2019, arXiv:1810.07888]

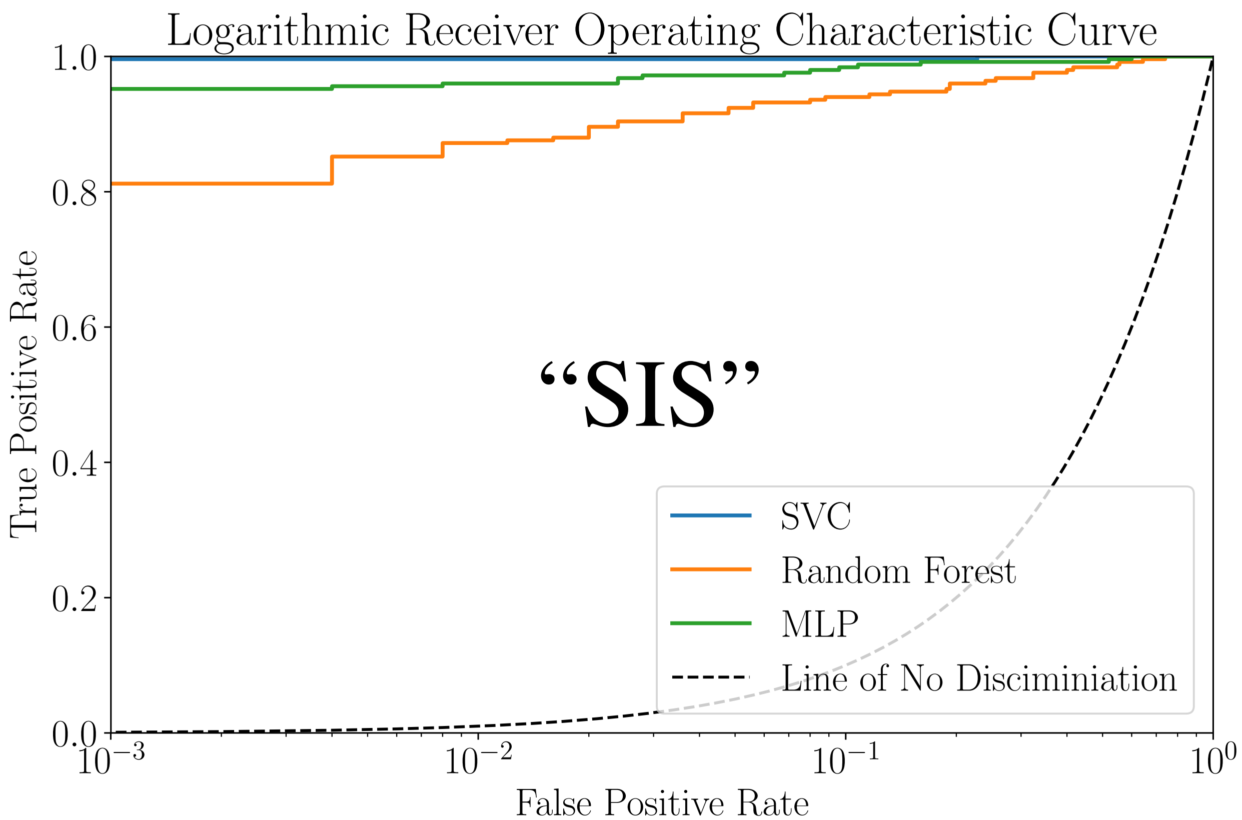
Classifiers	Hyper-parameters	Value
SVC	C	1000
	γ	5×10^{-7}
RFC	N_t	1000
	$N_{\rm mss}$	2
MLP	α	0.72
	$N_{layer\_neurons}$	1000

- ML algorithms
  - Support Vector Machine (SVC)
  - Random Forest (RF)
  - Multi-Layer Perceptron (MLP) ↔ Neural Network
- Confusion Matrix
  - Can say performance is good if
    - True Positive ~ 1, False Negative ~ 0, True Negative ~ 1, False Positive ~ 0



ROC Curve [Singh et al., AJUR 2019, arXiv:1810.07888]





- Area under Curve: $P(\mathrm{rank}_{\mathrm{signal}} > \mathrm{rank}_{\mathrm{noise}})$
  - PM
    - SVC: 0.995 / RF: 0.962 / MLP: 0.993
  - SIS
    - SVC: 0.999 / RF: 0.976 / MLP: 0.993

Summary

- Seems promising to classify lensed GWs from spectrograms of unlensed GWs even with simple ML models.
- For practical application, needs to
  - use LIGO/Virgo noise profile w/ proper restriction on observable SNR in the generation of input image samples and
  - use Qscan spectrogram which has been used in the representation of the chirp signal of detected GWs.

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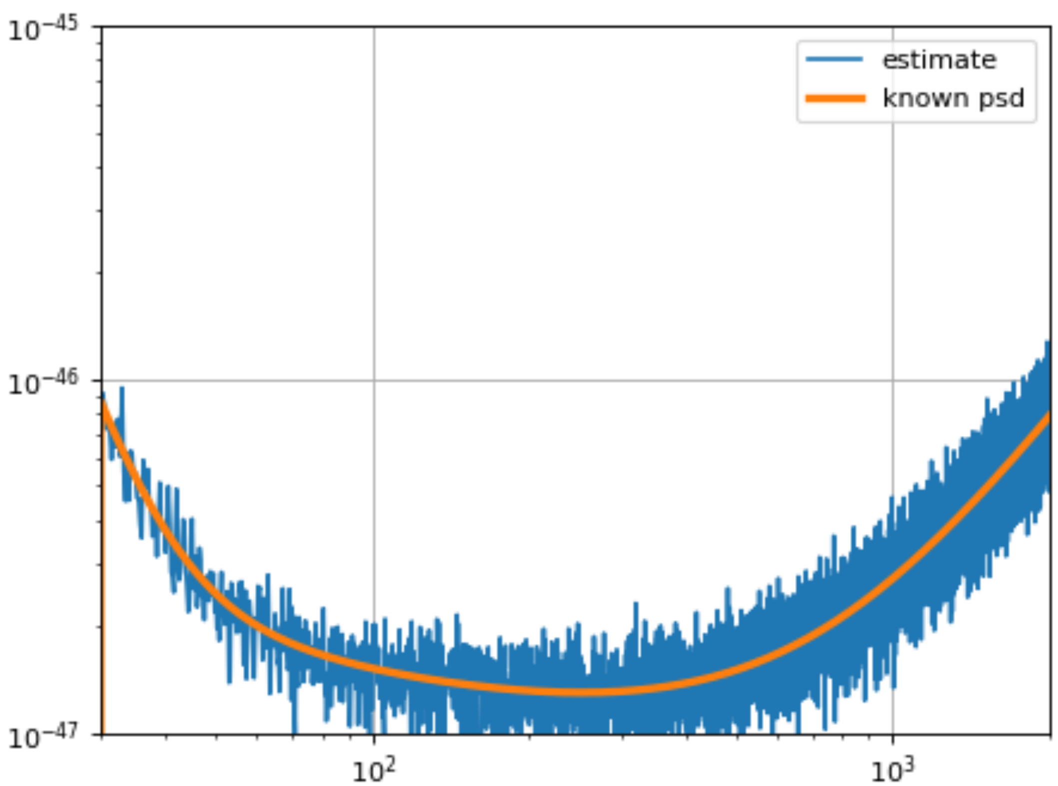

[from PyCBC documentation]

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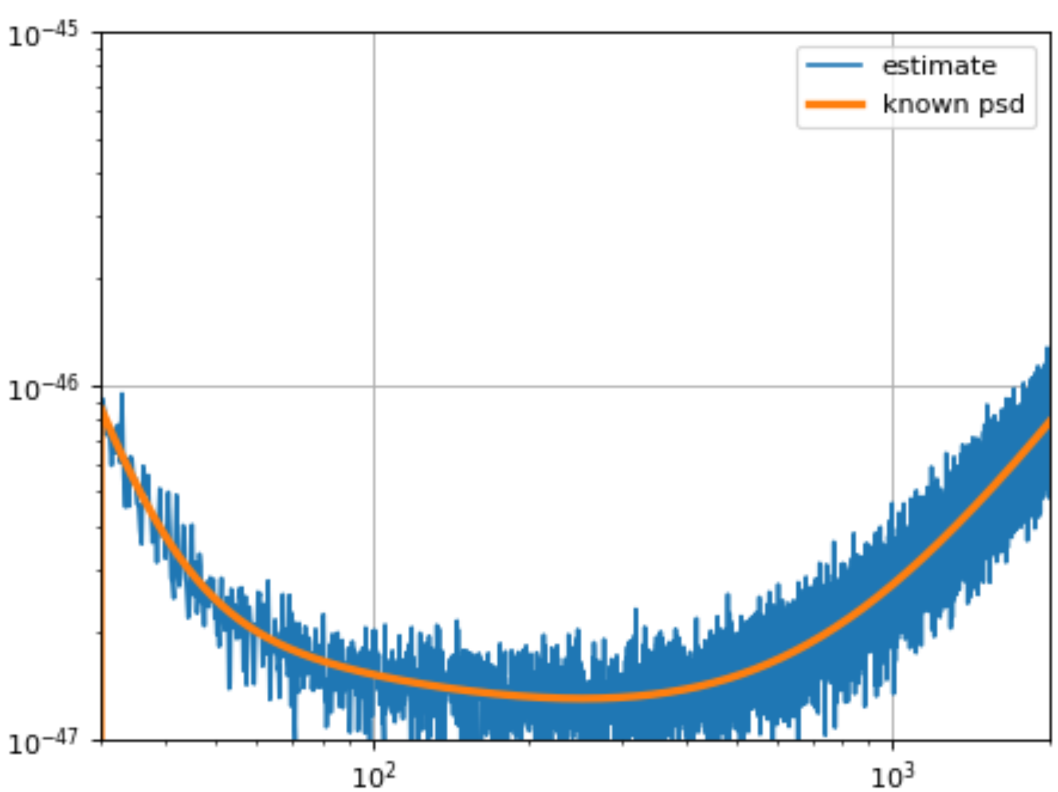

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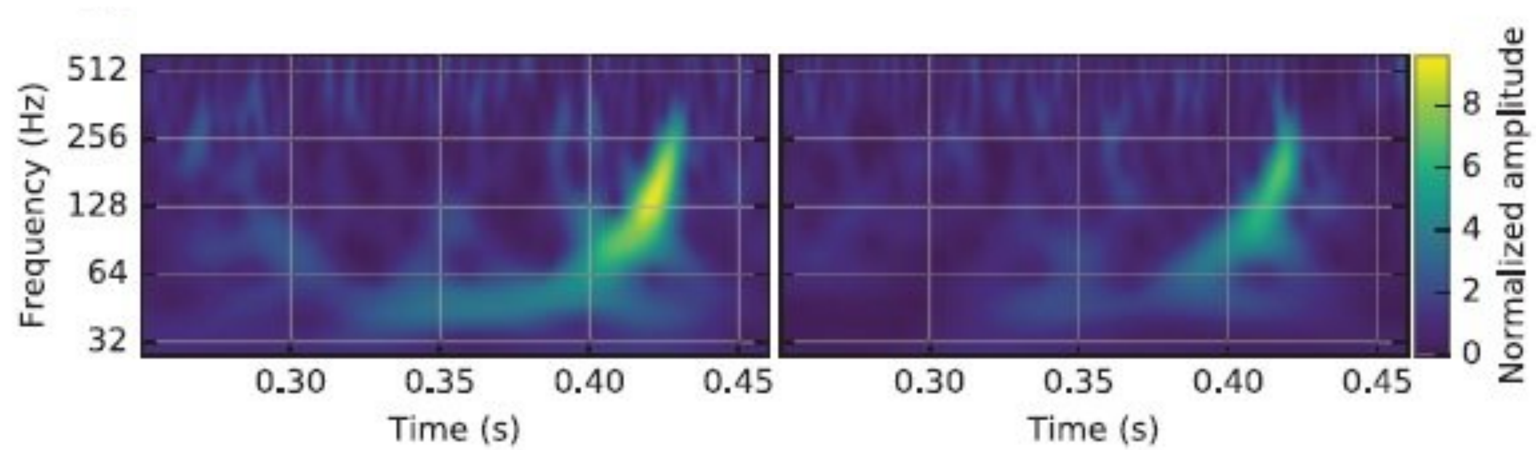
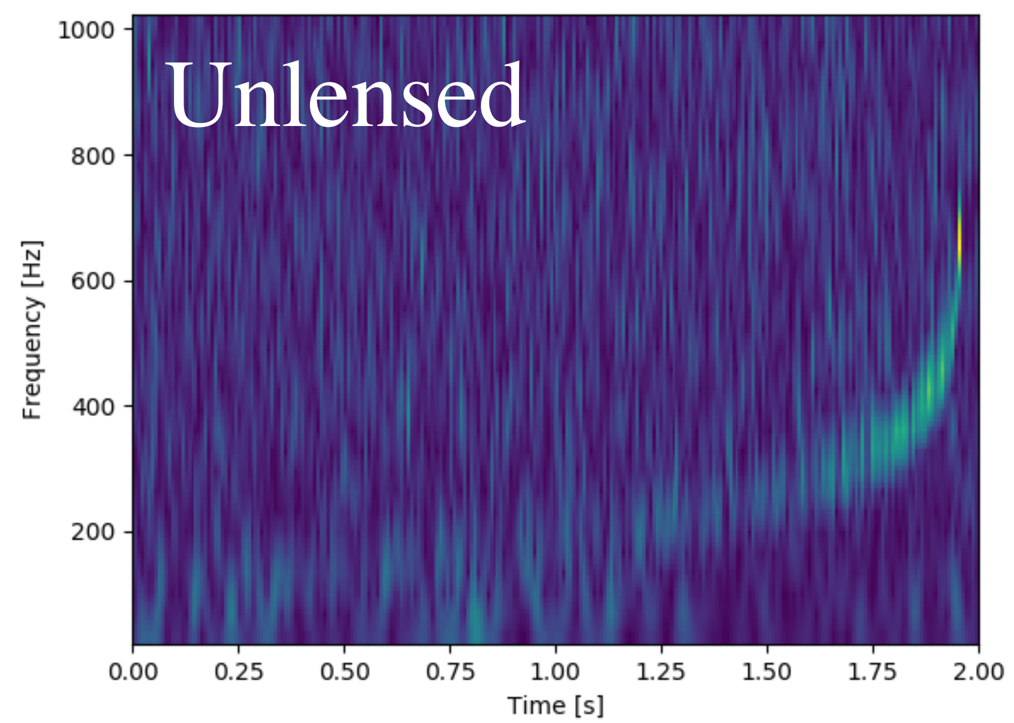
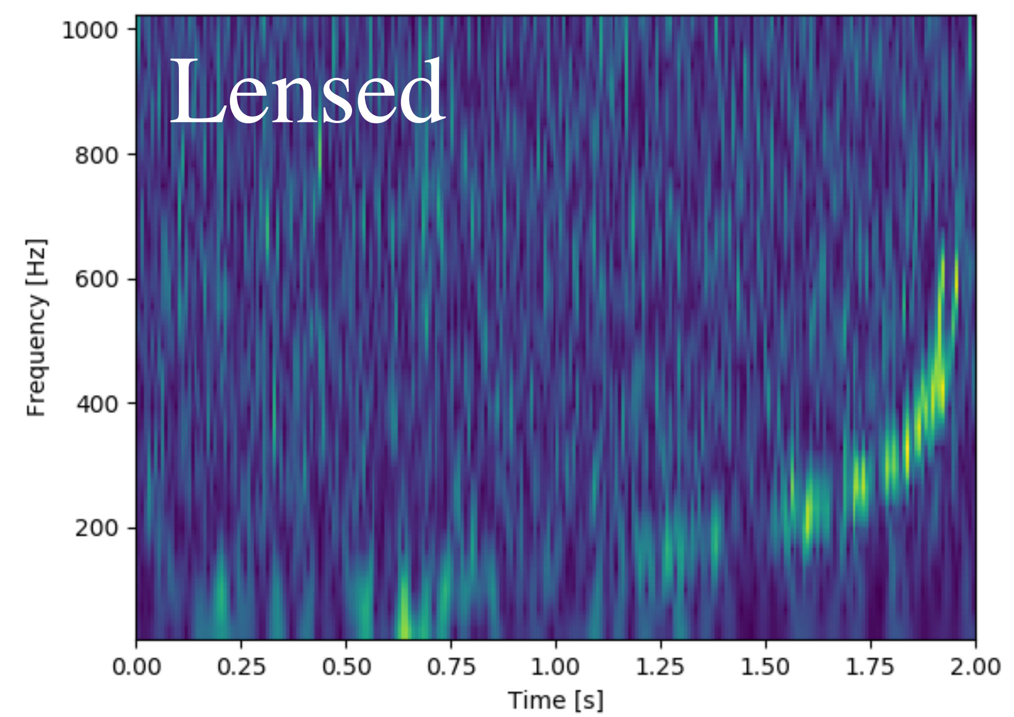


[from PRL **116**, 061102 (2016)]

Classification with Deep Learning w/ TensorFlow [Kim et al., *in progress*]

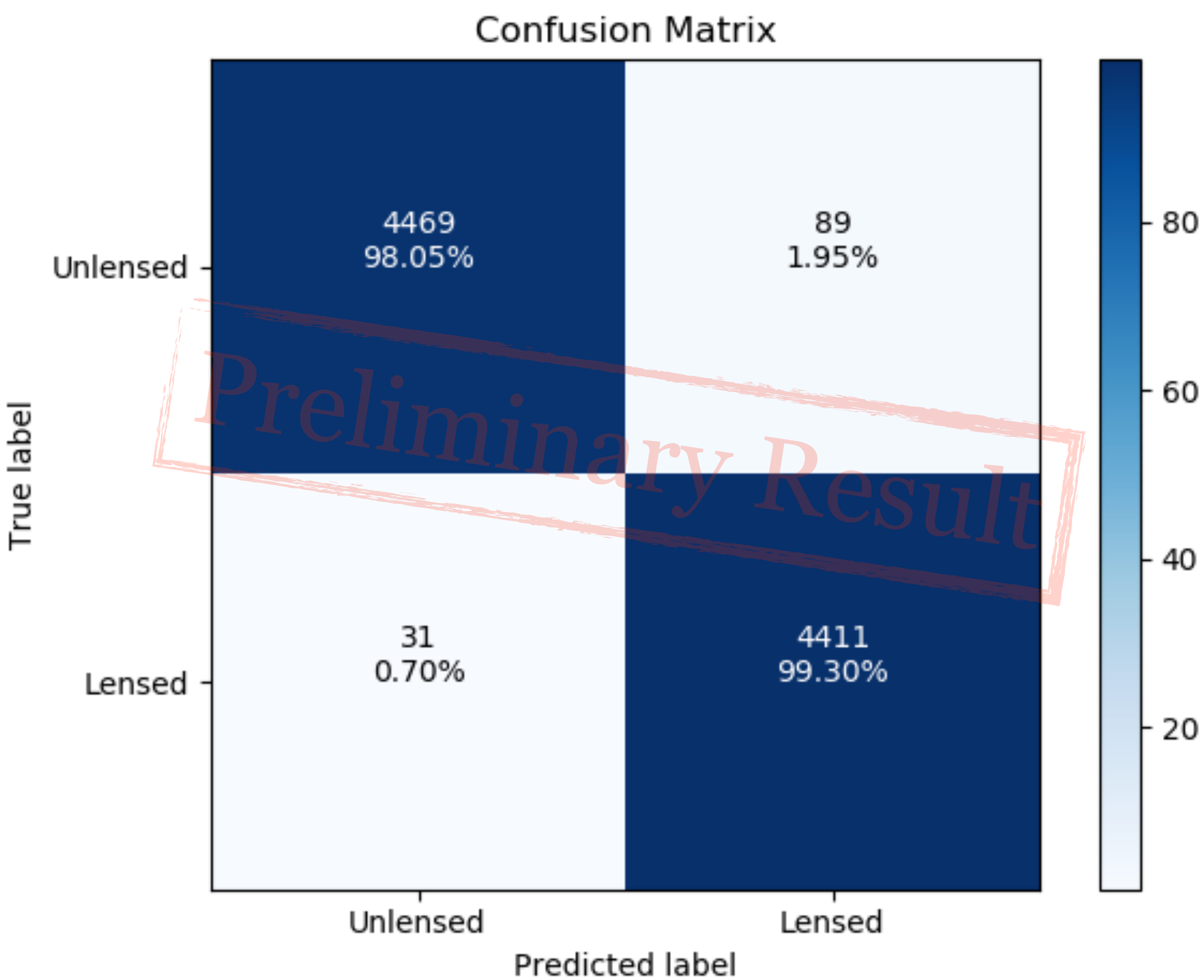
- Input data: 2-dim spectrogram using Qscan
  - # of samples: 45,000 (Training/Validation/Test: 27,000/9,000/9,000)
  - PM only
  - Parameters
    - m_1, m_2: 5 - 55 solar mass
    - D_L: 10 - 1000 Mpc
    - D_{LS}: 10 - 1000 Mpc
    - M_L: 10^3 - 10^5 solar mass
    - ε: 10^{-6} - 0.5 pc
    - redshift factor, z: $H_0 \times D$ / c (H_0 = 70 km/s/Mpc)
  - Noise: aLIGOZeroDetHighPower
    - $10 \leqq \mathrm{SNR} \leqq 50$ (c.f. ≤ 23.6 for O1&O2 BBHs in GWTC-1)

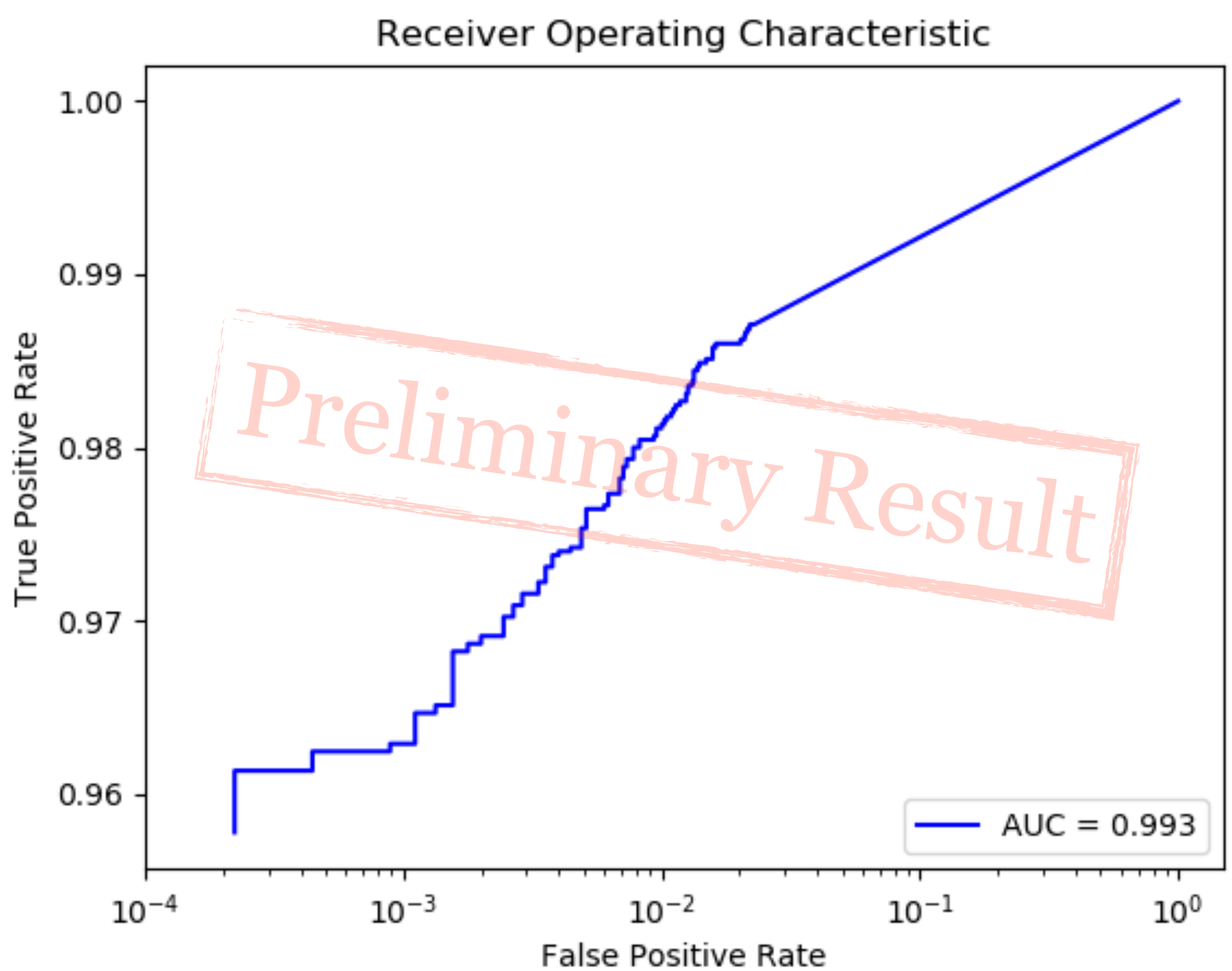




Classification Performance [Kim et al., *in progress*]

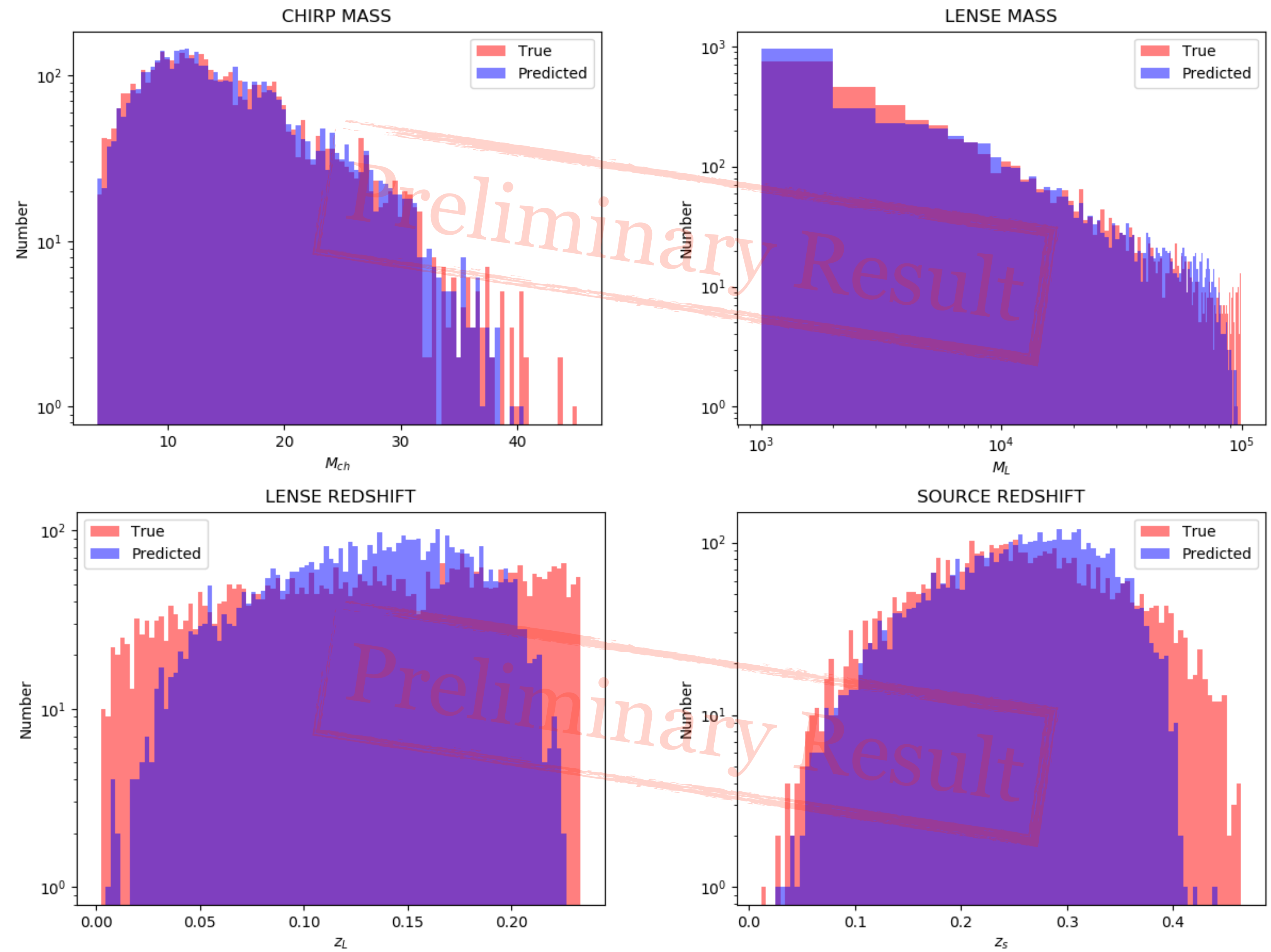
- DL model: VGG11
  - Activation function between hidden layers: Rectified Linear Unit (ReLU)
  - Output activation function: Sigmoid
  - Loss function for error measurement: Mean-Squared-Error





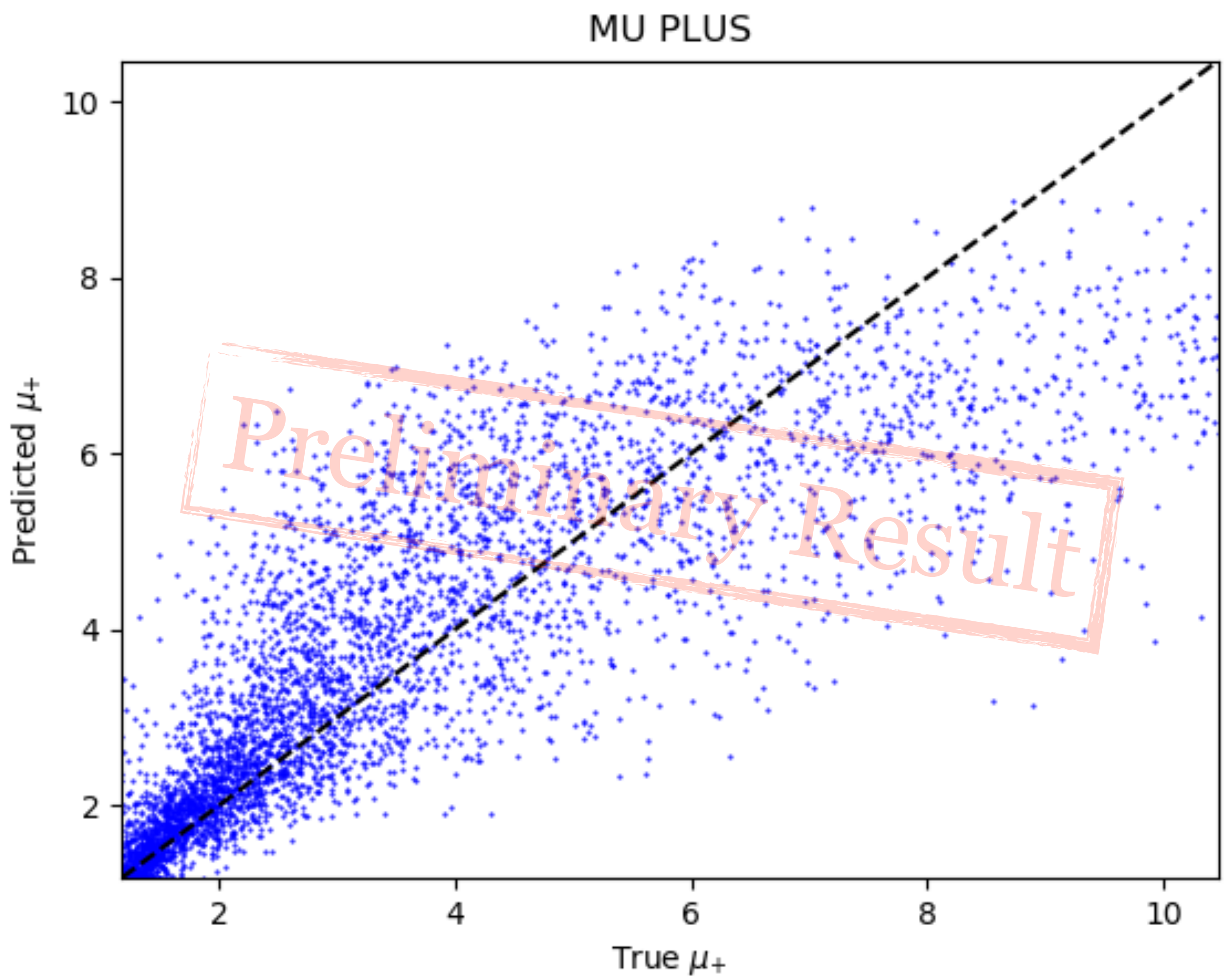
Regression with Deep Learning [Kim et al., *in progress*]

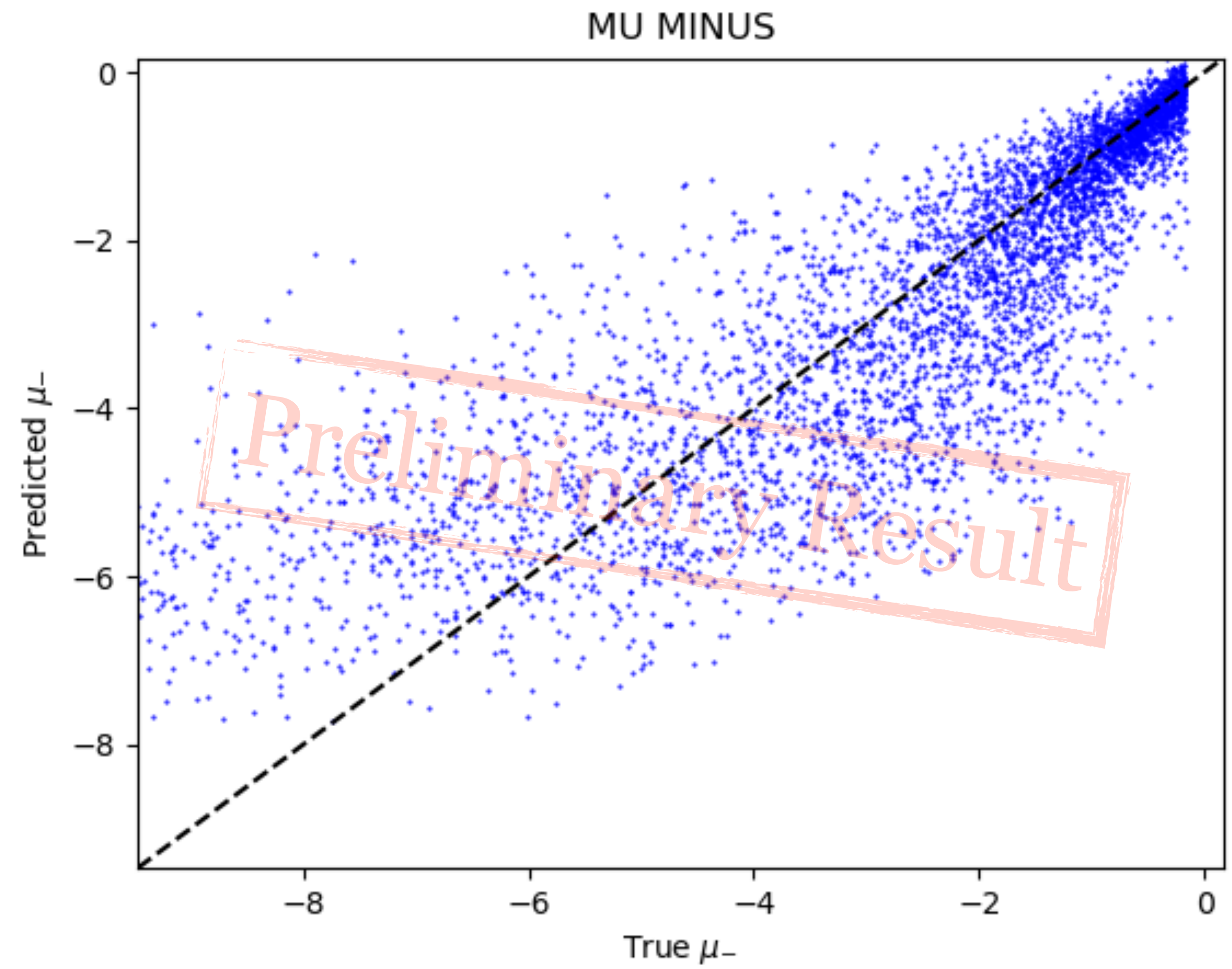
- Input data
  - The same 2-dim Qscan spectrograms used in classification
- DL model: VGG11 (the same with classification)
  - Activation function: ReLU (for both hidden layers and output layer)
  - Loss function: Smooth L1 loss



Comparison between True and Predicted Parameters [Kim et al., *in progress*]

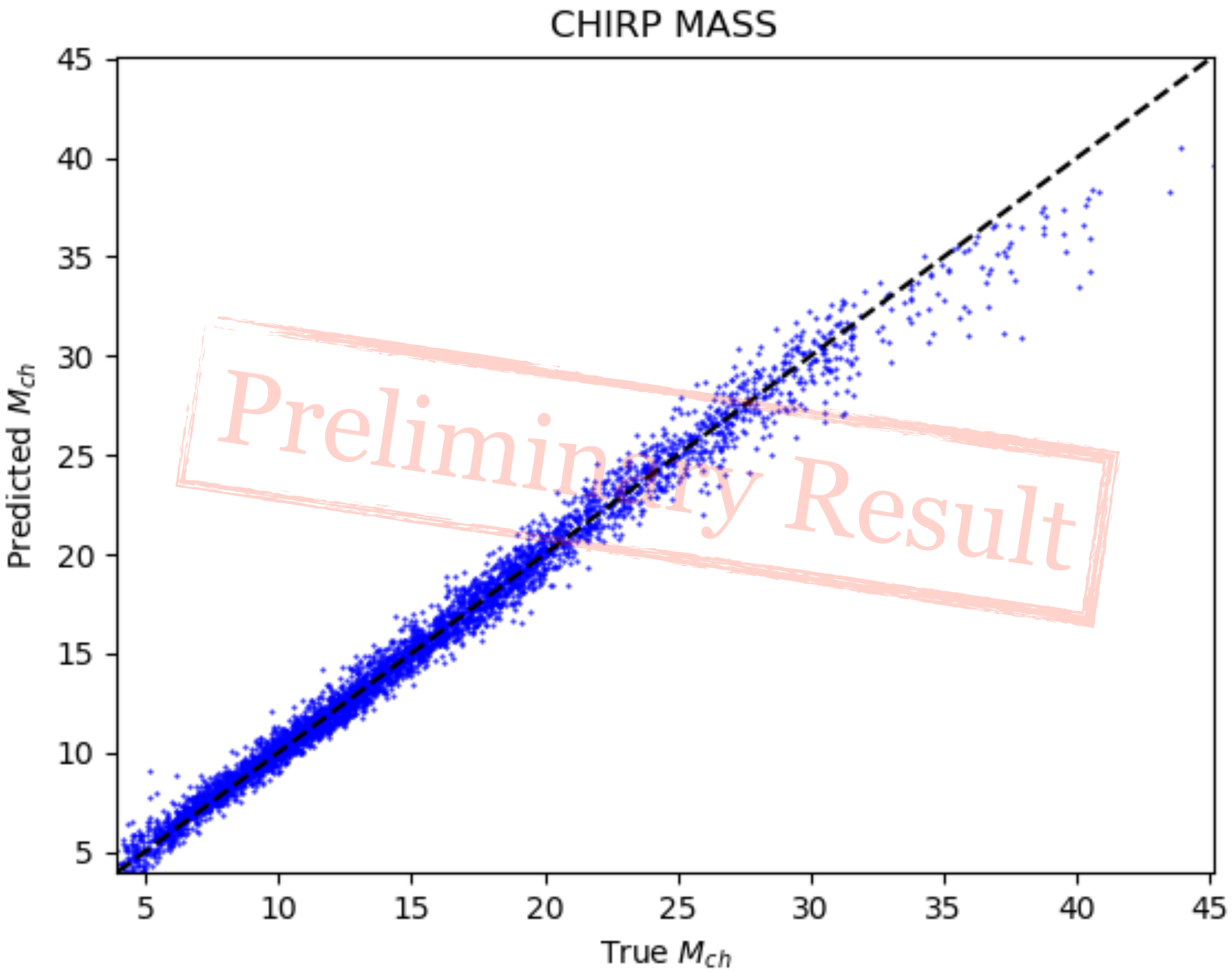
- Lensing factors

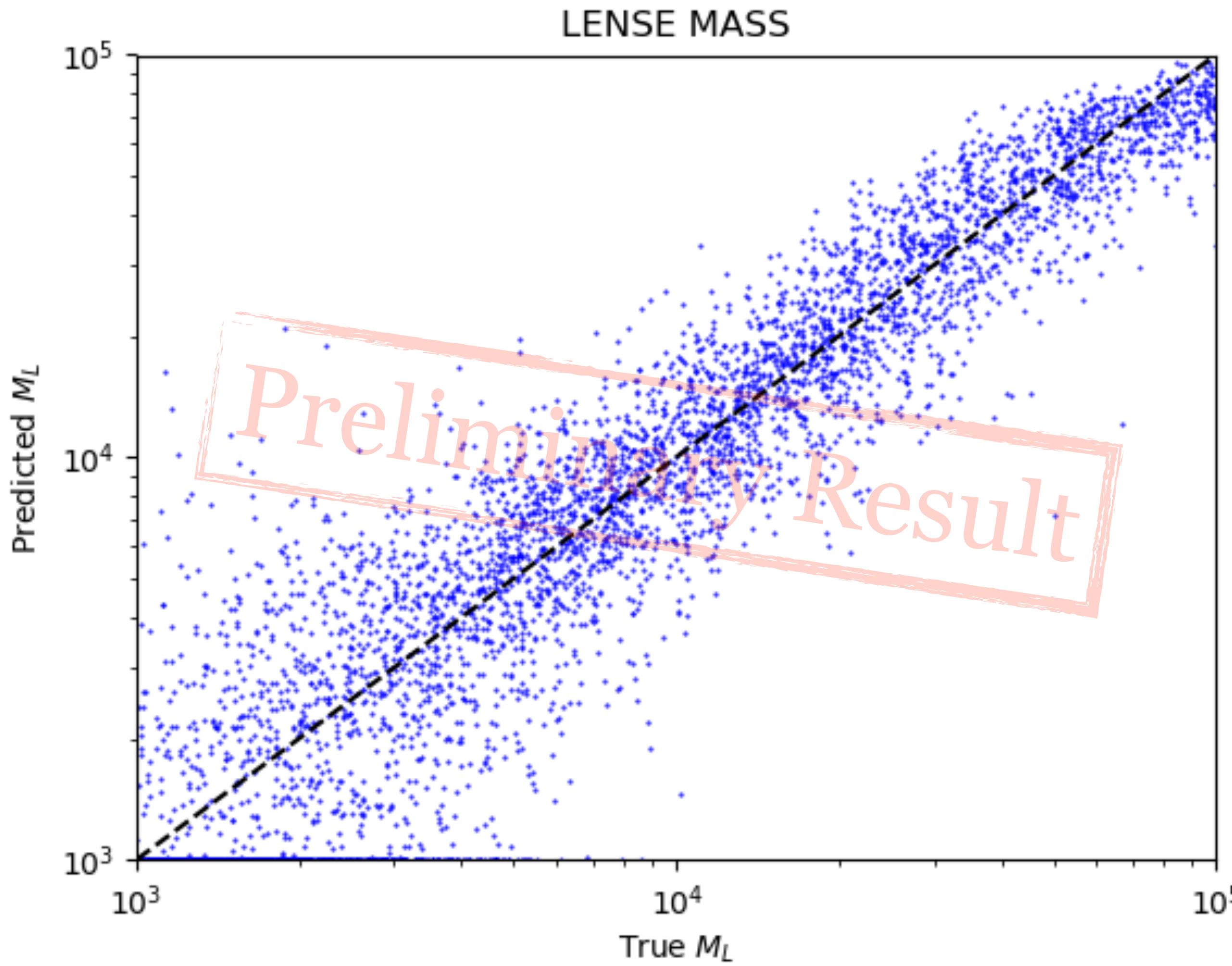




Comparison between True and Predicted Parameters [Kim et al., *in progress*]

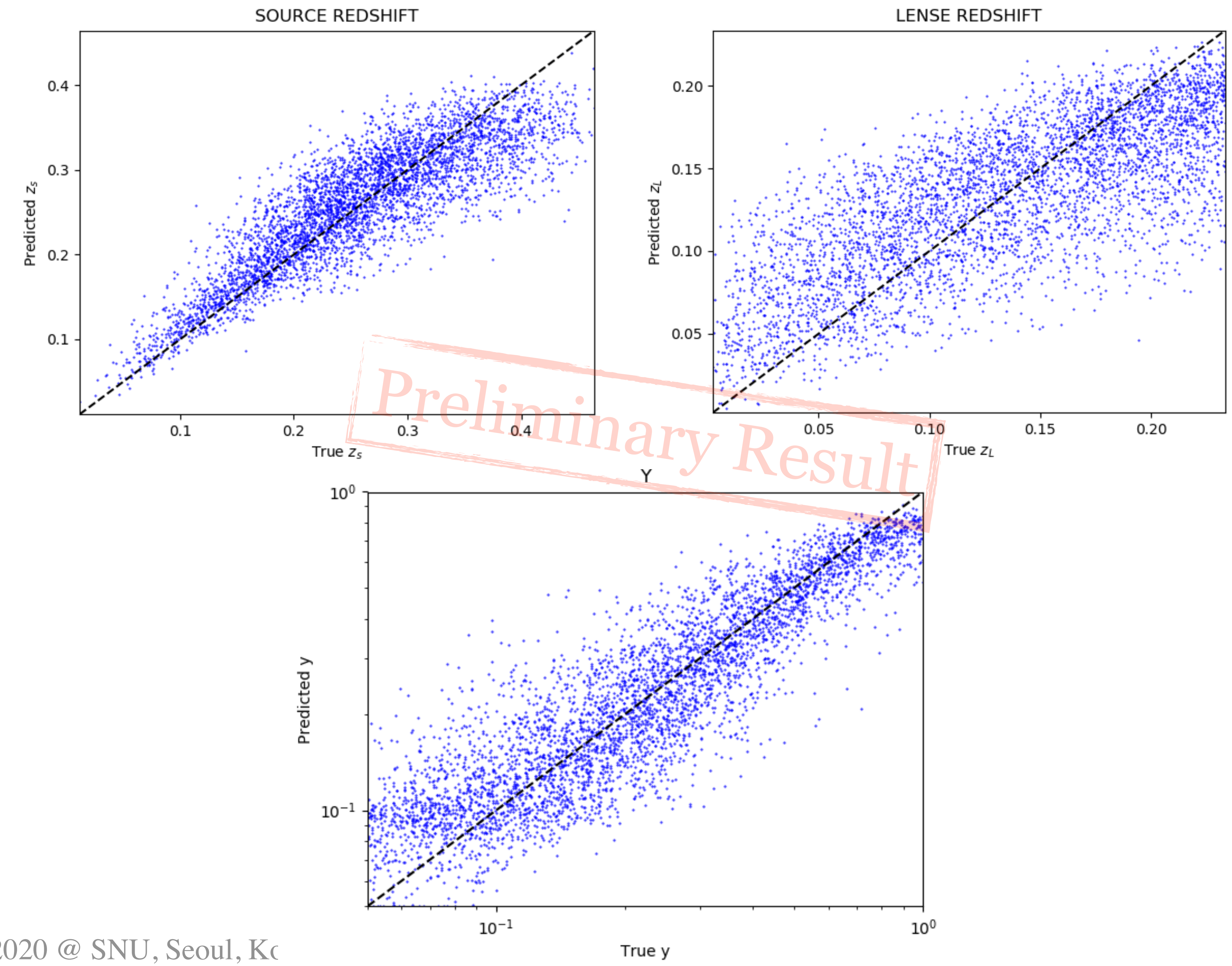
- Masses





Comparison between True and Predicted Parameters [Kim et al., *in progress*]

- Distances and Location



Summary

- DL is good at classification too.
- Seems possible to estimate lensing parameters from regression with DL.
  - Still needs to increase estimation power for poorly estimated parameters.
  - May needs to compare the estimation performance to a popular statistical method.
- Directions of future work
  - Tests SIS model too.
  - Studies the feasibility in wave optics.
  - Explores different physical parameter space with regarding astrophysical objects.
  - Builds a mature end-to-end framework from classification to regression.

Thank you! :)

Question?