Dark Odyssey 2020, Seoul National University 2020.1.6

Probing ultralight vector dark matter

K. Nomura, A. Ito and J. Soda,
``Pulsar timing residual induced by ultralight vector dark matter,''
arXiv:1912.10210 [gr-qc].
K. Nomura, A. Ito and J. Soda, work in progress.

Jiro Soda

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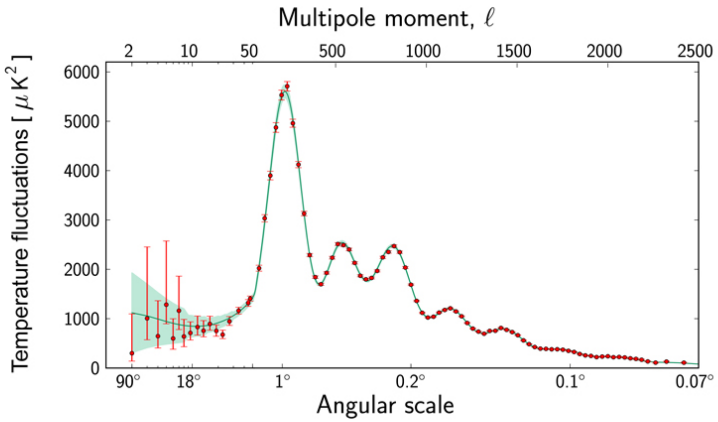
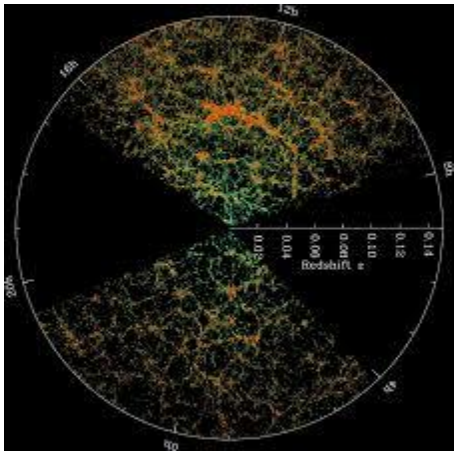
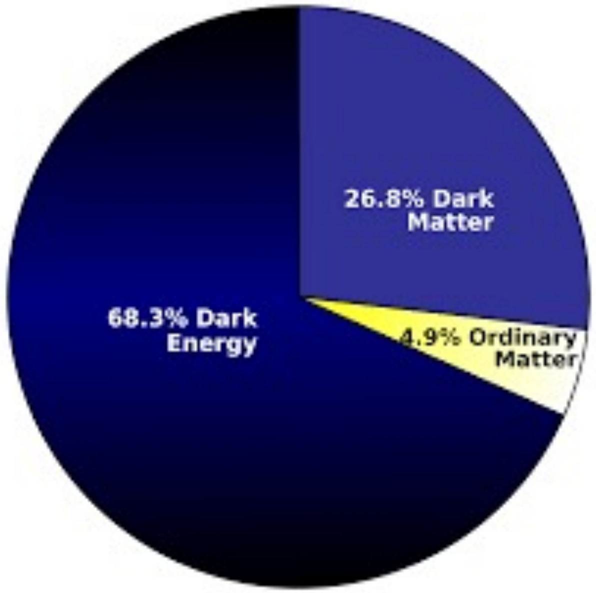
Dark matter exists!

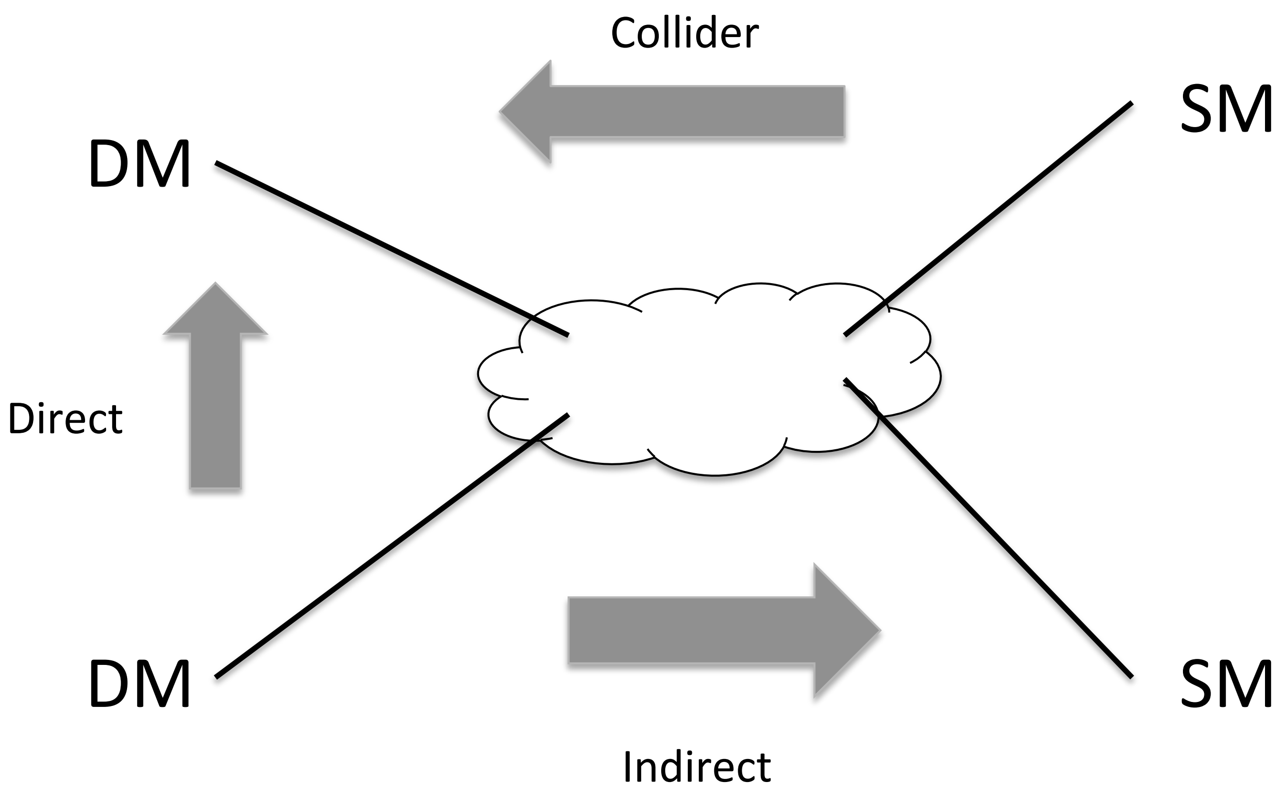






What is it?

DM hunting

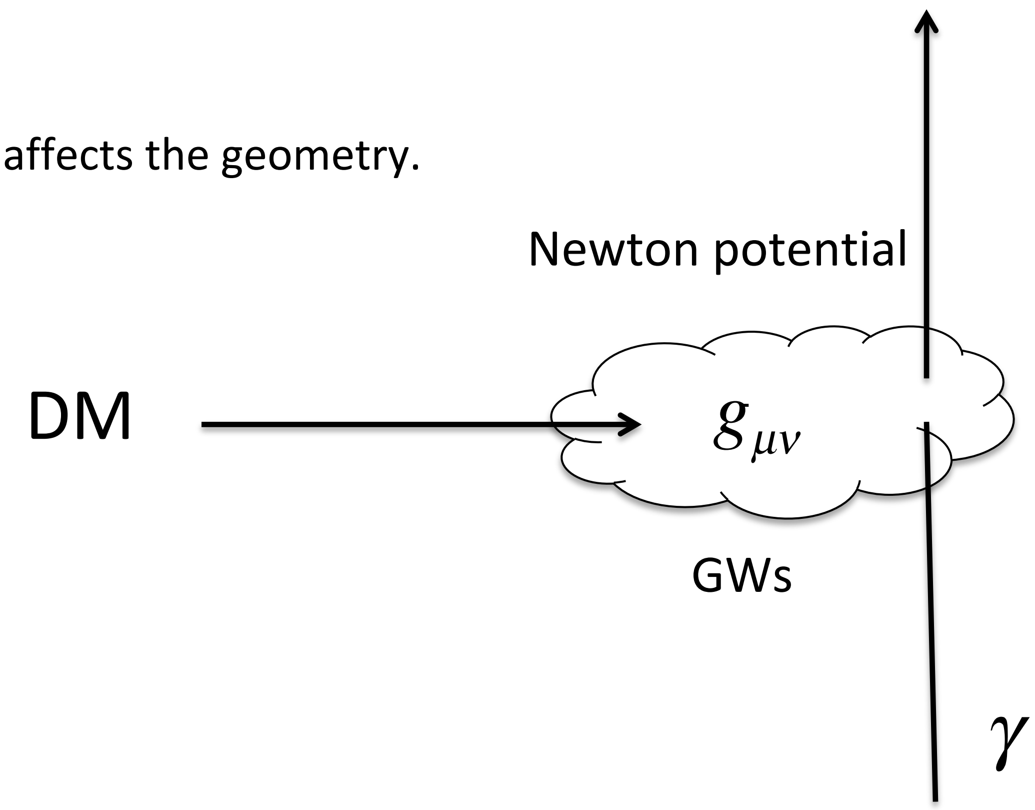


None of them has given any evidence of the DM so far.

DM hunting with gravity

We need to consider other dark matter candidates and …

DM affects the geometry.



As a DM candidate, we consider an ultralight particle.

Axion dark matter?

A typical example is an axion.

An ultralight scalar field (axion) dark matter erases extra structures on sub-galactic scales because of the effective quantum pressure. In fact, de Broglie wavelength of dark matter is given by

$$\lambda_{dB} = \frac{2\pi}{k} = \frac{2\pi}{mv} \approx 3.8\mathrm{kpc}\left(\frac{10^{-3}}{v}\right)\left(\frac{10^{-23}\mathrm{eV}}{m}\right)$$

Thus, the axion DM may resolve a cusp problem in the CDM,.

In string theory, there are a lot of ultralight scalar fields.
Thus, the ultralight axion dark matter has been intensively studied recently.

The axion would not be a unique possibility.
The higher spin DM can be considered.

What if an ultralight vector is DM?

Can a vector field survive during inflation?

Axion condensation

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + \frac{1}{2}m^2\phi^2\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

$m \ll H$ The scalar condensation can be formed.

Vector condensation?

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2A_\mu A^\mu\right]$$

$$\ddot{\bar{A}} + 3H\dot{\bar{A}} + \left(m^2 + 2H^2 + \dot{H}\right)\bar{A} = 0 \qquad \bar{A} = \frac{A}{a}$$

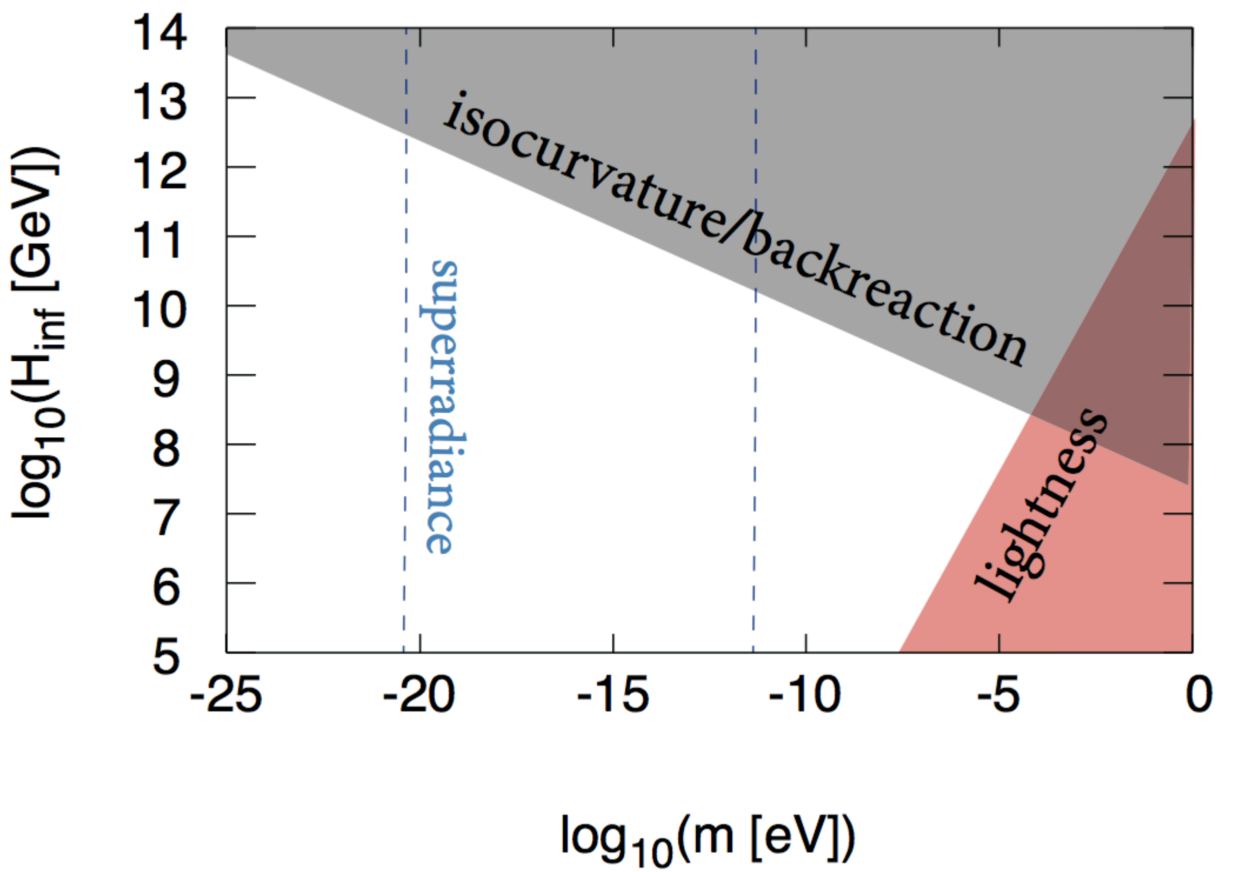
$m \ll H$ The vector condensation cannot be formed.

A vector field can survive!

Nakayama 2019

$$S=\frac{1}{2}\int d^4x\sqrt{-g}\,R+\int d^4x\sqrt{-g}\left[-\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}m^2A_\mu A^\mu\right]$$

The gauge kinetic function resolve the problem!



Statistical anisotropy in vector isocurvature

Nakayama has pointed out that there may be statistical anisotropy in isocurvature perturbations.

$$S_{DM} = \frac{\delta\rho_A}{\rho_A} \simeq \frac{1}{\rho_A}\frac{\partial\rho_A}{\partial A_i}\delta A_i \equiv n^i \delta A_i$$

This anisotropy can be probed with CMB.

It is interesting to see this feature with other methods.

One of our main purpose is to show that
we can probe this preferred direction with pulsar timing arrays.

Vector coherent oscillation

The vector model at present

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}m^2A_\mu A^\mu\right]$$

Occupation number
$$\frac{N}{\Delta x^3\Delta p^3}\approx\frac{n}{k^3}=\frac{\rho_{DM}}{m\,k^3}\approx 10^{90}\left(\frac{\rho_{DM}}{0.4\ \mathrm{GeV/cm^3}}\right)\left(\frac{10^{-23}\mathrm{eV}}{m}\right)^4$$

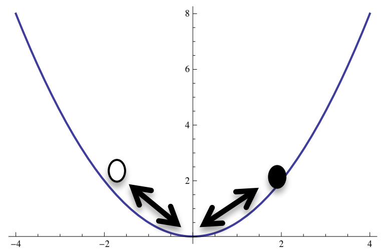
Since the occupation number is so high, classical field description is quite good.

We have a monochromatic frequency $E\approx m+\frac{1}{2}mv^2\approx m$ $\qquad v\approx 10^{-3}$

We can neglect the cosmic expansion on galactic scales.

$$\partial_\mu F^{\mu\nu}-m^2A^\nu=0 \qquad \partial_\nu A^\nu=0$$

$$\partial_\mu\partial^\mu A^\nu-m^2A^\nu=0 \qquad A^\nu=\left(0,\,0,\,0,\,A_z\right)$$



The axion is an coherently oscillating scalar field

$$A_z=A\cos\left(mt+\alpha\right)$$

Energy Momentum tensor of Vector DM

The energy density becomes

$$T_{00} = \rho_{DM} = \frac{1}{2}\dot{A}_z^2 + \frac{1}{2}m^2 A_z^2 = \frac{1}{2}m^2 A^2$$

The pressure is anisotropic

$$T_{xx} = T_{yy} = \frac{1}{2}\dot{A}_z^2 - \frac{1}{2}m^2 A_z^2 = -\frac{1}{2}m^2 A^2 \cos\left(2mt + \alpha\right)$$

$$T_{zz} = -\frac{1}{2}\dot{A}_z^2 + \frac{1}{2}m^2 A_z^2 = \frac{1}{2}m^2 A^2 \cos\left(2mt + \alpha\right)$$

The average value of the anisotropic pressure over the oscillation period is zero.
Hence, there is no anisotropy in the cosmological expansion.
Moreover, the vector field can be regarded as the dust matter on cosmological scales.

However, we need to discriminate the vector DM from CDM.
To this aim, we need to look into the effects of coherent oscillations.

Effects of ultralight vector DM on the metric

On the galactic scales, the background spacetime can be regarded as the Minkowski spacetime.

The perturbed metric takes the form

$$ds^2 = -(1-2\Psi)dt^2 + \left[(1+2\Psi)\delta_{ij} + h_{ij}\right]dx^i dx^j$$

By solving Einstein equations, we can deduce the time dependence of the metric.

$$\delta\Psi = -\frac{\pi G \rho_{DM}}{3m^2}\cos(2mt+2\alpha)$$

$$h_{ij} = \frac{8\pi G \rho_{DM}}{3m^2}\cos(2mt+2\alpha)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

How can we see this time dependence of the metric?

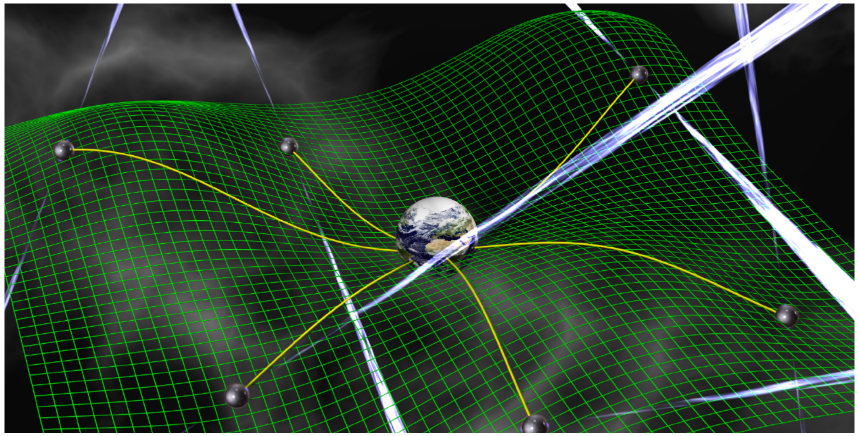
Probing vector DM with Pulsar Timing Arrays

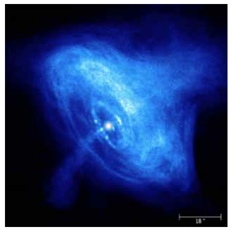
The geodesics of a photon emitted by a pulsar is affected by the oscillation of the DM

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu{}_{\nu\delta} k^\nu k^\delta = 0$$

The oscillation frequency around nHz can be probed by pulsar timing arrays.

$$f = \frac{\omega}{2\pi} = 5\times 10^{-9}\,\mathrm{Hz}\left(\frac{m}{10^{-23}\,\mathrm{eV}}\right)$$





Effect on the pulsar timing

The timing residual is given by the frequency shift $R = \int_0^t \frac{\Omega_0 - \Omega(t')}{\Omega_0} dt'$

The geodesic equation gives rise to the formula

$$z(t) = \frac{\Omega_0 - \Omega(t)}{\Omega_0} = \Psi(t, \mathbf{0}) - \Psi\left(t - |\mathbf{x}_p|, \mathbf{x}_p\right) + \frac{1}{2} n^i n^j \left[h_{ij}(t_0, \mathbf{0}) - h_{ij}\left(t - |\mathbf{x}_p|, \mathbf{x}_p\right) \right]$$

Pulsar timing residual

$$R = -\left[\frac{\pi G \rho_{DM}}{m^3} (1 + 2\cos\theta) \right] \sin\left(mD + \alpha(\mathbf{x}) - \alpha(\mathbf{x}_p)\right) \cos\left(2mt + \alpha(\mathbf{x}) + \alpha(\mathbf{x}_p) - mD\right)$$

This can be compared with the timing residual due to the axion DM

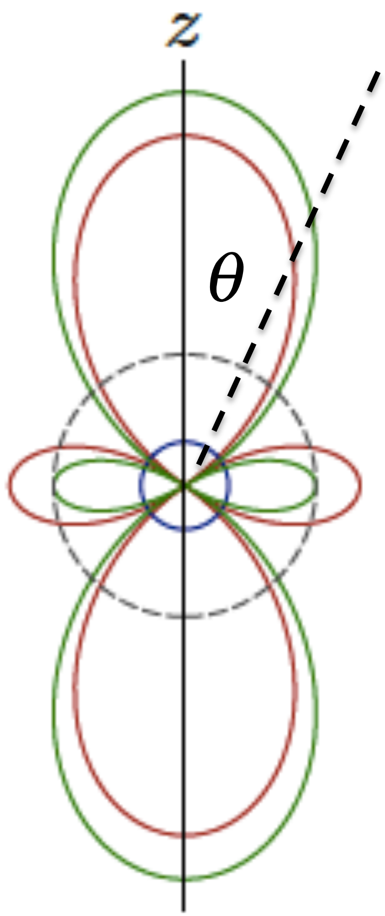
$$R = -\frac{\pi G \rho_{DM}}{m^3} \sin\left(mD + \alpha(\mathbf{x}) - \alpha(\mathbf{x}_p)\right) \cos\left(2mt + \alpha(\mathbf{x}) + \alpha(\mathbf{x}_p) - mD\right)$$

Khmelnitsky & Rubakov 2014

Angular dependence

Axion DM: grey dashed line

Vector DM: green line

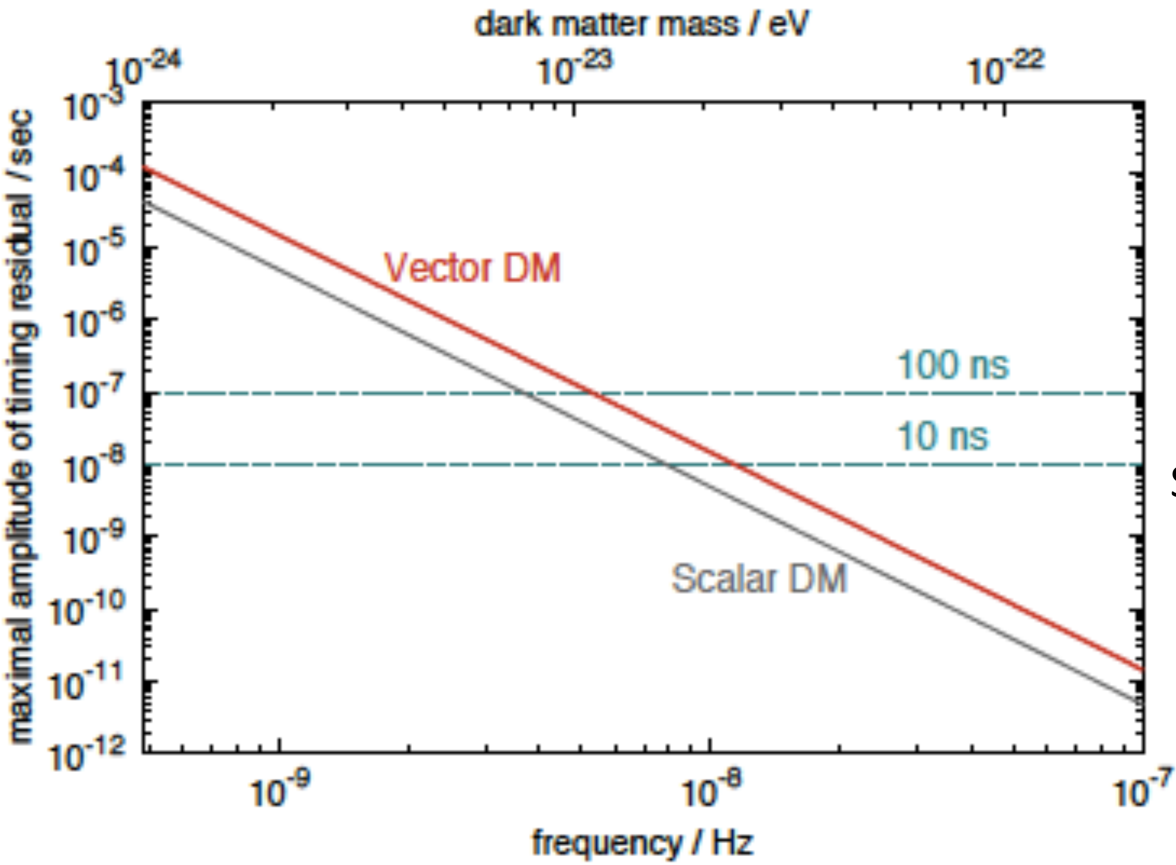


The direction
of the oscillation
of the vector DM

The signal from the vector DM is three times larger than that of axion DM.

There exists a nontrivial angular dependence.

Detectability



current sensitivity

SKA expected sensitivity

$$\max R = \frac{3\pi G \rho_{DM}}{m^3} = 1.3 \times 10^{-7} \left(\frac{\rho_{DM}}{0.4\ \mathrm{GeV/cm^3}} \right) \left(\frac{10^{-23}\,\mathrm{eV}}{m} \right)^3 \mathrm{sec}$$

Other directions

- Velocity of gravitational waves in the vector DM

  The vector and the graviton couple in the vector DM background.

  Hence, the dispersion relations are nontrivial.

- Gravitational waves from vector DM oscillations

  The vector DM is quite similar to a huge harmonic oscillator.

  Naively, we can expect

  $$h \approx \frac{G}{r}\left(\text{quadrapole kinetic energy}\right) \approx \frac{G}{r}\rho_{DM} L^3 \approx 10^{-13}\,? \qquad r = 10\ \text{Mpc}, \mathrm{L} = 10\ \text{kpc}, \text{m=}10^{-21}\text{eV}$$

- Structure formation with the vector DM

  The statistical anisotropy can play any role?

  It is interesting to consider the axion and the vector simultaneously.

- It is possible to consider the higher spin DM in a similar manner.

Summary

- We have shown that coherent oscillation of ultralight vector dark matter can be detected with pulsar timing arrays.
- Remarkably, the maximal amplitude is three times larger than that of axion DM.
- The pulsar timing residual has a non-trivial angular dependence.
- There are many interesting phenomena to be pursued.